Applied Mathematics

for

Industrial Hygiene and Safety

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Professional Safety Instruction

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About this Book: *Applied Mathematics for Industrial Hygiene and Safety* was developed to provide an in-depth review of math as it applies to industrial hygiene and safety. The focus is on equations used in certification exams for the CIH, CSP, and COHN-Safety credential. Developed as part of a continuing education program for busy industrial hygienists and safety professionals, this book also provides a valuable resource for those wishing to prepare for their registration exams.

We have tried to keep the formulas and variables as seen in the equation sheets used for the certification exams, but some changes have been made for clarity or consistency.

Although the focus has been on the application of common industrial hygiene and safety formulas, this book also shows the mathematical derivation of several important equations from basic principles. This approach was taken because of the importance in understanding, applying, and recalling the equations. Each formula or group of formulas includes a worked example. Common symbols, conversions, and constants are also included.

Finally, this book is a review of mathematics. The determination of the acceptability of the use of any equation or data presented in this book for addressing any industrial hygiene or safety issue is outside the scope of this work.

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Introductory Concepts

1 Introductory Concepts

1.1 Significant Figures

The significant figures (also called significant digits) of a number are those digits that carry meaning contributing to its precision. Digits that are not significant imply a false sense of precision and should not be reported. Calculators and spreadsheets routinely display more digits than those that are significant.

The following rules assist in deciding the correct number of significant figures.

Rule No.	Rule for Significant Figures
1	All non-zero digits (i.e., 1,2,3,4,5,6,7,8, and 9) are always significant.
2	All zeroes between non-zero numbers are always significant.
3	All zeroes which are simultaneously to the right of the decimal point <u>and</u> at the end of the number are always significant.
4	All zeroes which are to the left of a written decimal point <u>and</u> are in a number greater than or equal to 10 are always significant.

Note: One way to check rules 3 and 4 is to write the number in scientific notation. If you can eliminate any zeroes, then they are <u>not</u> significant.

Examples of Significant Figures

Number	# Significant Figures	Rule(s)
84,239	5	1
9.376	4	1
100.02	5	1,2,4
0.0005 (= 5 E-4)	1	1,4
2.3000	5	1,3
609.020	6	1,2,3,4
5,000,000 (= 5 E+6)	1	1
20.0 (= 2.00 E+1)	3	1,3,4

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1.1.1 Addition and Subtraction

When adding or subtracting numbers, count the number of decimal places to determine the number of significant figures. The answer cannot contain more places after the decimal point than the smallest number of decimal places in the numbers being added or subtracted.

Example: Add three number, 12.345678 + 9.8765 + 0.12

12.345678	(6 places after the decimal point)
+ 9.8765	(4 places after the decimal point)
+ 0.12	(2 places after the decimal point)
= 22.342178	(displays on calculator)
= 22.34	(rounded to 2 places in the answer)

Notice there are four significant figures in the answer.

1.1.2 Multiplication and Division

When multiplying or dividing numbers, count the number of significant figures. The answer cannot contain more significant figures than the number being multiplied or divided with the least number of significant figures.

Example: Multiply 98.765432 times 1.2345

98.765432	(8 significant figures)
x 1.2345	(5 significant figures)
= 121.9259258	(displayed on calculator)
= 121.93	(rounded to 5 significant figures)

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1.2 Scientific Notation

Scientific notation (sometimes called exponential notation) is a way of writing or displaying numbers in terms of a decimal number between 1 and 10 multiplied by a power of 10. Scientific notation numbers use the form:

$$a\,x10^{b} \qquad (1)$$

Scientific notation is typically used when numbers are too large or small to be conveniently written in standard decimal notation.

For example, Avogadro's number is the number of molecules in a mole of a substance. In scientific notation Avogadro's number is written as approximately 6.0225×10^{23} which is much easier than writing all those zeros.

1.3 Exponents and Radicals

Exponents and radicals are used extensively in the mathematics of safety and industrial hygiene. The following table summarizes the important rules for exponents and radicals.

Rule	Notes	Example
$a^n = a \cdot a \cdot a \cdot a \cdot a \ldots \cdot a$	*a* times itself n times	$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$
$a^0 = 1$	$a \neq 0$	$3.14^0 = 1$
$a^{-n} = \dfrac{1}{a^n}$	$a \neq 0$	$5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$
$a^n a^m = a^{n+m}$		$a^{-7}a^2 = a^{-7+2} = a^{-5}$
$\left(a^n\right)^m = a^{nm}$		$\left(a^3\right)^7 = a^{3 \cdot 7} = a^{21}$
$\dfrac{a^n}{a^m} = \begin{cases} a^{n-m} \\ \dfrac{1}{a^{m-n}} \end{cases}$	$for\,\dfrac{1}{a^{m-n}}, a \neq 0$	$\dfrac{a^n}{a^m} = \dfrac{a^2}{a^3} = a^{2-3} = a^{-1} = \dfrac{1}{a}$
$\left(ab\right)^n = a^n b^n$		$\left(ab\right)^{-7} = a^{-7}b^{-7}$
$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$b \neq 0$	$\left(\dfrac{a}{b}\right)^5 = \dfrac{a^5}{b^5}$
$\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n = \dfrac{b^n}{a^n}$	$a \neq 0$	$\left(\dfrac{a}{b}\right)^{-2} = \left(\dfrac{b}{a}\right)^2 = \dfrac{b^2}{a^2}$

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$(ab)^{-n} = \frac{1}{(ab)^n}$	$a \cdot b \neq 0$	$(ab)^{-3} = \frac{1}{(ab)^3}$
$\frac{1}{a^{-n}} = a^n$		$\frac{1}{a^{-9}} = a^9$
$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$	$a \neq 0$	$\frac{a^{-4}}{b^{-6}} = \frac{b^6}{a^4}$
$\left(a^n b^m\right)^k = a^{nk} b^{mk}$		$\left(a^2 b^{-3}\right)^4 = a^{2\cdot4} b^{-3\cdot4} = a^8 b^{-12}$
$\left(\frac{a^n}{b^m}\right)^k = \frac{a^{nk}}{b^{mk}}$	$b \neq 0$	$\left(\frac{a^2}{b^5}\right)^3 = \frac{a^{2\cdot3}}{b^{5\cdot3}} = \frac{a^6}{b^{15}}$
$\sqrt[n]{a} = a^{\frac{1}{n}}$	n is a positive integer > 1 and *a* is a positive real number	$\sqrt[3]{a} = a^{\frac{1}{3}}$
$\sqrt[n]{a^n} = a$	n is a positive integer > 1 and *a* is a positive real number	$\sqrt[5]{a^5} = a$
$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$	n is a positive integer > 1 and *a* and *b* are positive real numbers	$\sqrt[4]{ab} = \sqrt[4]{a}\sqrt[4]{b}$
$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	n is a positive integer > 1 and *a* and *b* are positive real numbers	$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Problem: Simplify the following expressions; provide answers with only positive exponents:

1. xy^{-2}
2. $\frac{a}{3b^{-5}}$
3. $\left(2x^{-3}y^4\right)^2$
4. $\left(-4a^2b^{-4}\right)^2\left(a^3b\right)^{-5}$
5. $\frac{n^{-2}m}{7m^{-4}n^{-3}}$

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6. $\left(\dfrac{12t^2s^{-8}}{3t^{-5}s}\right)^{-2}$

Solution:

1. $xy^{-2} = x\dfrac{1}{y^2} = \dfrac{x}{y^2}$
2. $\dfrac{a}{3b^{-5}} = \dfrac{1}{3}a\dfrac{1}{b^{-5}} = \dfrac{ab^5}{3}$
3. $\left(2x^{-3}y^4\right)^2 = 2^2x^{-6}y^8 = \dfrac{2^2y^8}{x^6} = \dfrac{4y^8}{x^6}$
4. $\left(-4a^2b^{-4}\right)^2\left(a^3b\right)^{-5} = \left(-4\right)^2a^4b^{-8}a^{-15}b^{-5} = 16a^{-11}b^{-13} = \dfrac{16}{a^{11}b^{13}}$
5. $\dfrac{n^{-2}m}{7m^{-4}n^{-3}} = \dfrac{m^4n^3m}{7n^2} = \dfrac{m^5n}{7}$
6. $\left(\dfrac{12t^2s^{-8}}{3t^{-5}s}\right)^{-2} = \left(\dfrac{4t^2t^5}{s\cdot s^8}\right)^{-2} = \left(\dfrac{4t^7}{s^9}\right)^{-2} = \dfrac{4^{-2}t^{-14}}{s^{-18}} = \dfrac{s^{18}}{16t^{14}}$

1.4 Logarithm Functions

Logarithmic functions are used in several areas of safety and industrial hygiene, including those related to sound and noise as well as radiation.

The definition of the logarithm function is:

If b is any number such that $b > 0$ and $b \neq 1$ and $x > 0$ then,

$$y = \log_b x \tag{2}$$

This is read as "log base b of x" and is equivalent to:

$$b^y = x \tag{3}$$

Although the base (b) can be any number complying with the definition, the most common logarithm functions are the *common* and *natural* logarithms,

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$$\text{common logarithm: } \log x = \log_{10} x \tag{4}$$

$$\text{natural logarithm: } \ln x = \log_e x \tag{5}$$

where $e = 2.71828...$

Note the natural logarithm is written ln, not log.

The following table reviews the important rules related to logarithmic functions.

	Rule	Notes
1	$\log_b 1 = 0$	$b^0 = 1$
2	$\log_b b = 1$	$b^1 = b$
3	$\log_b b^x = x$	
4	$b^{\log_b x} = x$	
5	$\log_b (xy) = \log_b x + \log_b y$	x>0 and y>0
6	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$	x>0 and y>0
7	$\log_b (x^r) = r \log_b x$	x>0 and y>0
8	If $\log_b x = \log_b y$ then $x = y$	x>0 and y>0

Note that there is no rule for breaking up a logarithm for the sum or difference of two terms, i.e.,

$$\log_b (x + y) \neq \log_b x + \log_b y \tag{6}$$

$$\log_b (x - y) \neq \log_b x - \log_b y \tag{7}$$

Here's a simple example of a logarithm function:

$$\log_6 216 = 3 \quad \textit{just as} \quad 6^3 = 216$$

Many other examples of logarithms are presented in Section 9 on Sound and Noise and Section 10 on Radiation.

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1.5 Absolute Value Equations

In mathematics, the absolute value (or modulus) of a real number is the numerical value of that number without regard to its sign (i.e., it is considered positive and no sign is shown). Absolute value is shown by a vertical bar on each side of the number:

$$|a| = \begin{cases} a \text{ if } a \geq 0 \\ -a \text{ if } a < 0 \end{cases} \tag{8}$$

Problem: What is the absolute value of -5?

Solution:

$$|-5| = 5$$

1.6 Quadratic Formula

A quadratic equation is a second-order polynomial equation with a single variable, *x*, in the form:

$$ax^2 + bx + c = 0 \tag{9}$$

where $a \neq 0$ (when $a = 0$ the equation becomes linear). Because Equation (9) is a second-order polynomial equation, the fundamental theorem of algebra guarantees that it has two solutions. This is found by the quadratic formula, which is derived by completing the squares as follows:

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \tag{10}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2} \tag{11}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a} \tag{12}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{13}$$

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± Note the ± symbol in the above equations. This symbol means the term after the ± sign is both added, and subtracted, from the term before the ± sign.

Problem: Solve $x^2 + 3x - 4 = 0$

Solution: By reviewing equation (10) we see for this equation a=1, b=3, c=-4, and substituting those values into equation (13) yields:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{25}}{2} = \frac{-3 \pm 5}{2}$$

$$x = \frac{-8}{2}, \frac{2}{2} = -4, 1$$

1.7 Boolean Algebra

Boolean algebra can be thought of as the algebra of events and states. Boolean algebra is important in the construction and mathematical evaluation of event trees, such as fault trees, particularly when a large number of events are related in some manner.

The most common rules for Boolean algebra are shown in the accompanying table. Boolean algebra assumes A, B, and C (etc.) are logical states that can have the values 0 (false) and 1 (true). Although the nomenclature used may vary depending on preference, the following are typical examples of Boolean algebra formats, where "+" means OR, "·" means AND, and A' means NOT A.

Rules for Boolean Algebra

Addition	Multiplication	Rule
A + A = A	A · A = A	identity
A + 0 = A	A · 0 = 0	operation with 0
A + 1 = 1	A · 1 = A	operation with 1
A + A' = 1	A · A' = 0	complement
A + B = B + A	A · B = B · A	commutative law
A + (B + C) = (A + B) + C	A · (B · C) = (A · B) · C	associative law
A + (A · B) = A	A · (A + B) = A	absorption
A + (B · C) = (A + B) · (A + C)	A · (B + C) = (A · B) + (A · C)	distributive law

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Consider the following example. Assume you want to evaluate the probability of an uncontrolled fire occurring at some location. When constructing an event tree (e.g., a fault tree) to evaluate this scenario, two high level events are required. A fire must occur, and the fire must not be controlled (notice the "and" in the statement – both events are required). Each of these events can be broken down further. The failure to control the fire can be broken down to two other events; failure of automatic methods (e.g., sprinklers) and failure of manual methods (e.g., fire department) – again note the "and." With regard to sprinkler system failure, this could be due to the fire pump failing to start or the pre-action valve failing to open. Notice the 'or' here, either event would lead to failure; both are not required.

This is a very simple example, but you can see how such an analysis could quickly generate a very large number of events. Boolean algebra allows you to quantify the events and rank the importance of contributing events.

Problem: Resolve the following Boolean expression.

$$\left(A+B\right)\cdot\left(A+B'\right)$$

Solution: First, we can expand the statement to find:

$$\left(A+B\right)\cdot\left(A+B'\right)=A\cdot A+A\cdot B'+A\cdot B+B\cdot B'$$

and

$$A\cdot A+A\cdot B'+A\cdot B+B\cdot B'=A+A(B+B')+0$$

since $A\cdot A=A$ and $B\cdot B'=0$ and then

$$A+A(B+B')+0=A+A=A$$

since $B+B'=1$ and $A+A=A$ we find the above expression resolves to A.

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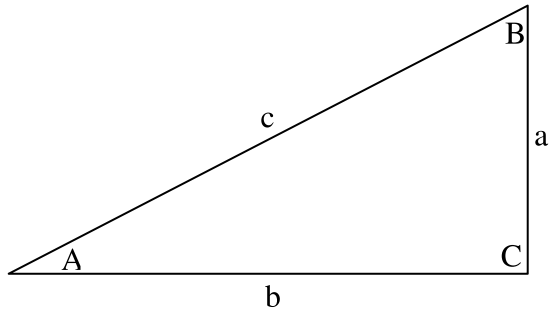
1.8 Trigonometric Functions



1.8.1 Right Triangles

For a right triangle (i.e., one with angle C=90°) equations (14) through (16) are true:

$$\sin A = a / c \tag{14}$$

$$\cos A = b / c \tag{15}$$

$$\tan A = a / b \tag{16}$$

1.8.2 Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C \tag{17}$$

Note when C = 90° (i.e., for a right triangle) equation (17) reduces to the Pythagorean Theorem, equation (18).

1.8.2.1 Pythagorean Theorem

$$a^2 + b^2 = c^2 \tag{18}$$

1.8.3 Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \tag{19}$$

Problem: You walk about 50 feet away from the base of a water tank. From that location it appears the top of the water tank is about 60 degrees above the ground. About how high is the top of the water tank?

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Solution: Reviewing the triangle diagram above, we see we have angle "A" and side "b" and want side "a". Equation (16) can be written to allow us to estimate the height of the tank (side "a" in the diagram).

$$a = b \cdot \tan A = 50 \tan 60^o = 86.6\,\text{feet}$$

So we can estimate our water tank is about 90 feet high. To understand the possible error with our estimate, we need to know the error with the horizontal measurement and the angle used.

1.9 Useful Equations for Geometric Shapes

1.9.1 Perimeter

$$\text{Triangle: } P = a + b + c \tag{20}$$

$$\text{Rectangle: } P = 2L + 2W \tag{21}$$

$$\text{Square: } P = 4s \tag{22}$$

$$\text{Circle: } C = \text{circumference} = \pi d = 2\pi r \tag{23}$$

1.9.2 Area

$$\text{Triangle: } A = \frac{1}{2} bh \tag{24}$$

$$\text{Rectangle: } A = LW \tag{25}$$

$$\text{Square: } A = s^2 \tag{26}$$

$$\text{Circle: } A = \pi r^2 = \frac{\pi d^2}{4} \tag{27}$$

$$\text{Parallelogram: } A = bh \tag{28}$$

$$\text{Trapezoid: } A = \tfrac{1}{2} h\left(b_1 + b_2\right) \tag{29}$$

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1.9.3 Volume

$$\text{Rectangular solid: } V = LWH \tag{30}$$

$$\text{Cube: } V = s^3 \tag{31}$$

$$\text{Sphere: } V = \tfrac{4}{3}\pi r^3 \tag{32}$$

$$\text{Circular cylinder: } V = \pi r^2 h \tag{33}$$

$$\text{Circular cone: } V = \tfrac{1}{3}\pi r^2 h \tag{34}$$

$$\text{Regular pyramid: } V = \tfrac{1}{3}s^2 h \tag{35}$$

1.9.4 Surface Area

$$\text{Rectangular solid: } SA = 2LW + 2LH + 2WH \tag{36}$$

$$\text{Cube: } SA = 6s^2 \tag{37}$$

$$\text{Sphere: } SA = 4\pi r^2 \tag{38}$$

$$\text{Right circular cylinder: } SA = 2\pi r^2 + 2\pi rh \tag{39}$$

$$\text{Right circular cone: } SA = \pi r^2 + \pi rl \tag{40}$$

$$\text{Regular pyramid: } SA = s^2 + 2sl \tag{41}$$

Problem: A cylindrical tank with a diameter of 3 feet stands 6 feet tall. What is the volume of the tank in cubic-feet? How many gallons of liquid can this tank hold? Assuming the tank is used for water and another for acetone, how many pounds of water or acetone can each tank hold? Note: Assume water weight 62.4 lbs/ft^3.

Solution: First, we can calculate the volume of the tank in cubic feet using equation (33).

$$V = \pi r^2 h = \pi\left(\frac{3}{2}\,\text{ft}\right)^2 \cdot 6\,\text{ft} = 42.4\,\text{ft}^3$$

We can convert to gallons using the conversion 1 ft^3 = 7.481 gallons,

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$$42.4\,\text{ft}^3\left(\frac{7.481\,\text{gal}}{1\,\text{ft}^3}\right)=317.2\,\text{gallons}$$

We can find the capacity of the tank in pounds of water by converting volume to pounds of water as follows:

$$42.4\,\text{ft}^3\left(62.4\frac{lbs}{ft^3}\right)=2645.8\,\text{lbs}_{\text{H}_2\text{0}}$$

To find the weight for acetone, we can use the specific gravity. We can find the specific gravity from data on its MSDS sheet. MSDS typically list the specific gravity of acetone as 0.79 (water = 1.0). Since we know the weight in water, we simply multiply that by the specific gravity for acetone:

$$\left(2645.8\,\text{lbs}_{\text{H}_2\text{0}}\right)(0.79)=2090.2\,\text{lbs}_{\text{acetone}}$$

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Statistics

2 Statistics

2.1 Arithmetic Mean

The arithmetic mean, often referred to simply as the average, is a method to derive the central tendency of a sample space. The term "arithmetic mean" is preferred because it helps distinguish it from other averages, such as the geometric mean. The arithmetic mean is calculated as follows:

$$\bar{X} = \frac{X_1 + X_2 + \ldots + X_n}{n} \tag{42}$$

where

$\bar{X}$ = arithmetic mean of n items

X_n = value of nth item

n = total number of items to be averaged

The Ellipsis (…) Equation (42) contains a common symbol, the ellipsis (…). In mathematics, an ellipsis is often used to indicate "and so on." Equation (42) can be described as reading “add X_1 and X_2 *and so on* for as many items as you have, and then divide by the number of item you have.” It is common in mathematics to indicate the number of items by the variable n.

2.2 Geometric Mean

The geometric mean, is similar to the arithmetic mean except that the sample numbers are multiplied and then the nth root of the resulting product is taken, as shown here,

$$GM = \sqrt[n]{(X_1)(x_2)\ldots(X_n)} \tag{43}$$

where

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GM = geometric mean of n items

n = total number of items to be averaged

X_n = value of nth item

The following equation is simply another form of the geometric mean equation above.

$$GM = 10^{\frac{\sum_{i=1}^{n}(\log X)}{n}} \tag{44}$$

where

GM = geometric mean of n items

X_n = value of nth item

n = total number of items to be averaged

i = count

Σ Notation Mathematical formulae often require the addition of many variables. The summation notation, indicated by a capital Greek sigma, is the common form of shorthand used to give a concise expression for a sum of the values of a variable. For example:

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \ldots + x_n$$

Problem: Several air samples are taken to determine the airborne concentration of a process solvent. The following concentrations (in ppm) are found: 51, 76, 49, 79, and 36. Calculate the arithmetic and geometric means.

Solution: To calculate the arithmetic mean, we use equation (42) and for the geometric mean we use equation (43).

$$\bar{X} = \frac{X_1 + X_2 + \ldots + X_n}{n} = \frac{51+76+49+79+36}{5} = \frac{291}{5} = 58.2$$

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$$GM = \sqrt[5]{(51)(76)(49)(79)(36)} = 55.8$$

The results demonstrate that all *averages* are not the same. The selection of the mean equation will depend on the application of the data. Typically, if numbers are to be added, use an arithmetic mean. If values are to be multiplied, use a geometric mean. For example, if a investment return yielded 12, 17 and 14 percent over a three year period, the appropriate average would be the geometric mean since the gains are compounded (i.e., multiplied).

2.3 Standard Deviation

The standard deviation of a data set is the square root of its variance. Standard deviation is a widely used measure of the variability or dispersion; that is it shows how much variation there is from the "average." A low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values.

The sample standard deviation is the most common estimator for a "standard deviation." It is an adjusted version (i.e., N-1) and is typically denoted by as s or SD.

Another estimator for the standard deviation is not adjusted (i.e. N) and is typically denoted by *σ*. It has a uniformly smaller mean squared error than the sample standard deviation. It provides the maximum-likelihood estimate when the population is normally distributed. But this estimator, when applied to smaller samples, tends to be too low.

The two are typically expressed as follows:

When N-1 this is a "Sample Standard Deviation" (usually written SD):

$$SD = \sqrt{\frac{\sum_{i=1}^{n} \left(\bar{x} - x_i\right)^2}{n-1}} \qquad (45)$$

This can also be written as:

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$$SD = \sqrt{\frac{\sum_{i=1}^{n}\left(x^2\right)}{n-1}} \tag{46}$$

where $x = \overline{x} - x_i$

When n is used unmodified this is a "Standard Deviation" (usually written as σ):

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(\overline{x} - x_i\right)^2}{n}} \tag{47}$$

This can also be written as:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(x\right)^2}{n}} \tag{48}$$

where $x = \overline{x} - x_i$

N or n In some formulas for standard deviation, you may see *n* written as an upper case *N*. In this application they are simply used to denote the total number of items being evaluated, so either form is acceptable.

Problem: Several air samples are taken to determine the airborne concentration of a process solvent. The following concentrations (in ppm) are found; 51, 76, 49, 79, 36. Calculate the sample standard deviation and the standard deviation.

Solution: We can use equations (45) and (47). Also note the following term is the same in each equation:

$$\sum_{i=1}^{n}\left(\overline{x} - x_i\right)^2$$

We also know the arithmetic mean ($\overline{X} = 58.2$) from the sample problem above.

Next, we can calculate term above as shown in the following table:

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x_i	$\overline{x} - x_i$	$\left(\overline{x} - x_i\right)^2$
51	7.2	51.84
76	-17.8	316.84
49	9.2	84.64
79	-20.8	432.64
36	22.2	492.84
$\sum_{i=1}^{n}\left(\overline{x} - x_i\right)^2$		1378.8

We can now solve for the sample standard deviation:

$$SD = \sqrt{\frac{1378.8}{5-1}} = 18.57$$

And the standard deviation:

$$\sigma = \sqrt{\frac{1378.8}{5}} = 16.61$$

2.4 Geometric Standard Deviation

The geometric standard deviation describes how spread out a set of numbers is whose average is characterized by a geometric mean. In safety and industrial hygiene applications related to particle size distributions, the geometric standard deviation (of a lognormal distribution) is easily determined by dividing the mass median particle diameter by the particle size at the 15.87 percent probability *or* by dividing the particle size at the 84.13 percent probability by the mass median particle diameter. These two equations are shown here:

$$GSD = \frac{50\%tile\ value}{15.87\%tile\ value} \tag{49}$$

$$GSD = \frac{84.13\%tile\ value}{50\%tile\ value} \tag{50}$$

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where

GSD = geometric standard deviation

Problem: Particle sample data are plotted on logarithmic graph paper, and the resulting plot reveals the average particle size is 10µm, and 84.13% of the cumulative particle mass is below 20µm and 15.87% of the cumulative particle mass is below 5µm. Calculate the geometric standard deviation of the samples.

Solution: Both equations (49) and (50) should provide the same determination.

$$GSD = \frac{50\%tile\ value}{15.87\%tile\ value} = \frac{10\mu m}{5\mu m} = 2.0$$

$$GSD = \frac{84.13\%tile\ value}{50\%tile\ value} = \frac{20\mu m}{10\mu m} = 2.0$$

2.5 Coefficient of Variation

The coefficient of variation is a measure of relative variation of a set of normally-distributed values; it is calculated as follows:

$$CV = \frac{SD}{\overline{X}} \tag{51}$$

where

CV = coefficient of variation (see following equation), percent in decimal format

SD = the sample standard deviation (see equation (45))

$\overline{X}$ = the arithmetic mean (see equation (42))

Problem: Several air samples are taken to determine the airborne concentration of a process solvent. The following concentrations (in ppm) are found; 51, 76, 49, 79, 36. Calculate the coefficient of variation.

Solution: The formula for the coefficient of variation is given in equation (51). For the data set in the problem, the arithmetic mean and sample standard deviation were derived in the sample problems above ($\overline{X}$ = 58.2, and SD = 18.57).

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$$CV = \frac{SD}{\bar{X}} = \frac{18.57}{58.2} = 0.312 = 31.2\%$$

2.6 Cumulative Error

In some cases, the individual errors associated with various steps in a measurement can be quantified. However, the total cumulative error is not just a simple summation of the individual errors, rather the cumulative error is defined by the following expression:

$$E_c = \sqrt{E_1^2 + E_2^2 + \ldots + E_n^2} \tag{52}$$

where

E_c = cumulative error

E_n = individual error of item n

n = total number of error items

Problem: Consider a case in which sampling and analytical errors (SAE) are used to account for a margin of error before measured exposures are determined to exceed the total airborne contaminant limit. Assume the total air sampling error factor accounts for three uncontrollable variances; 1) air pump performance (CV_P), 2) variability of the deposit area on the filter (CV_D) and 3) variability of the laboratory analysis (CV_A). These values are $CV_P = 0.04$, $CV_D = 0.5$, $CV_A = 0.07$. What is the total variance?

Solution: To determine CV_{total}, the individual components are determined separately and then combined according to the cumulative error formula, equation (52):

$$CV_{total} = \sqrt{CV_P^2 + CV_D^2 + CV_A^2} = \sqrt{0.04^2 + 0.5^2 + 0.07^2} = 0.506$$

2.7 Sampling and Analytical Error

All sampling and analytical methods have some degree of uncertainty. The total uncertainty depends on the combined effects of the contributing uncertainties inherent in sampling and analysis. Uncertainty in sampling results has historically been called Sampling and Analytical Error (SAE) by OSHA. It can be calculated as follows:

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$$SAE = 1.645 \cdot CV_{total} \tag{53}$$

where

SAE = Sampling and Analytical Error

1.645 = a constant that is a 95-percent 1-tailed confidence coefficient

CV_{total} = coefficient of variation (see equation (51)), percent in decimal format

Problem: Based on the total coefficient of variation just calculated, what is the sampling and analytical error for the method used (95% confidence)?

Solution:

$$SAE = 1.645 CV_{total} = 1.645(0.506) = 0.833$$

2.8 Student's t-Test

Any statistical test that uses the t-distribution can be called a t-test. One of the most common is Student's t-test. Student's t-test is used to compare the means of two samples. The shape of the t-distribution depends on the number of degrees of freedom. The degrees of freedom for a t-test is the total number of observations in the groups minus 2, or n_1+n_2-2.

These statistics can be used to carry out either a one-tailed test or a two-tailed test.

Once a t value is determined, a p-value can be found using a table of values from Student's t-distribution (See Table in Section 13). If the calculated p-value is below the threshold chosen for statistical significance (frequently the 0.05 level), then the null hypothesis is rejected in favor of the alternative hypothesis.

The following equation is used for t-tests:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SD_{pooled}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \tag{54}$$

where

t = the test statistic

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x_1 = mean of sample 1

x_2 = mean of sample 2

SD_{pooled} = pooled standard deviation (see following equation)

n_1 = number of measurements in sample set 1

n_2 = number of measurements in sample set 2

Tails? Generally, when you conduct a test of statistical significance, you are given a probability (p-value) in the output. Also, if your test statistic is symmetrically distributed (such as a t-distribution), you can select a one-tailed test or a two-tailed test. Most references on statistical tests will recommend that if there is any doubt, a two-tailed test should be done; that is, select your p-values from a two-tailed table. However, if you have a table of one-tailed data (e.g., the CSP examination reference t-distribution table), simply multiply the probability value by 2 and use the data from that column. For example, data for a two-tailed p-value of 0.1 is the same as a one-tailed p-value of 0.05 (i.e., 2*0.05 = 0.1). See the T-Distribution Table in Section 13.

2.9 Pooled Standard Deviation

The pooled standard deviation is used in the above t-test equation.

$$SD_{pooled} = \sqrt{\frac{(n_1 - 1)SD_1^2 + (n_2 - 1)SD_2^2}{n_1 + n_2 - 2}} \tag{55}$$

where

SD_{pooled} = pooled standard deviation

SD_1 = standard deviation for sample set 1

SD_2 = standard deviation for sample set 2

n_1 = number of measurements in sample set 1

n_2 = number of measurements in sample set 2

The following equation is used for an independent one-sample *t*-test.

$$t = \frac{\bar{X} - \mu}{SD}\sqrt{n-1} = \frac{\bar{X} - \mu}{\sigma}\sqrt{n} \tag{56}$$

where

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t = the test statistic

$\bar{X}$ = mean of sample

μ = mean of the population

SD = sample standard deviation

σ = standard deviation

n = sample size

Note that these can be written as:

$$t = \frac{\bar{X} - \mu}{\frac{SD}{\sqrt{n-1}}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \tag{57}$$

Problem: Two shifts at a factory each have 8 employees working at a time. The work requires repetitive motions, so short breaks are encouraged. You are asked to conduct an analysis of the breaks taken to ascertain if there is a significant difference between the two shifts. The student's t-test is a good tool since you are comparing two similar data sets. An initial assessment reveals the following data on the number of breaks taken, along with the totals, average and standard deviation:

	Group 1	Group 2
	5	8
	7	1
	5	4
	3	6
	5	6
	3	4
	3	1
	9	2
Total	40	32
Average	5	4
SD	2.14	2.56

Solution: With the data above, you can use equations (55) and (54) to determine the t-test value.

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$$SD_{pooled} = \sqrt{\frac{(n_1 - 1)SD_1^2 + (n_2 - 1)SD_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(7)(2.14)^2 + (7)(2.56)^2}{8 + 8 - 2}} = 2.36$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SD_{pooled}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{5 - 4}{2.36\sqrt{\frac{1}{8} + \frac{1}{8}}} = 0.847$$

Now, going to the table of t-distributions (see Section 13), we see for 14 degrees of freedom (i.e., 16-2) and with a probability of 0.05 (two tails), t must be at least 2.145. Therefore we conclude the difference in breaks is not significant.

2.10 Normal Distribution Z Score

The number of standard deviations from the mean is called the *z-score*. One of the most useful applications of the normal distribution Z score is being able to determine the exact proportion of data that falls above and below that score. They are found by the formula:

$$z = \frac{X - \mu}{\sigma} \tag{58}$$

where

z = number of standard deviations between X and μ

X = value to be evaluated

μ = mean of the population (x in equation (42) above)

σ = standard deviation of the population

A negative Z-score means that the original score was below the mean. A positive Z-score means that the original score was above the mean. Z-scores are typically used in conjunction with standard normal curve data tables (see Section 13). The following example will demonstrate how z-scores are commonly used.

Problem: Several air samples are taken to determine the airborne concentration of a process solvent. The following concentrations (in ppm) are found; 51, 76, 49, 79, 36. Assuming a normal distribution, what is the probability of a reading greater than 80?

Solution: The formula for the z-score is given in equation (58). For the data set in the

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problem, the arithmetic mean and standard deviation were derived in the sample problems above ($\mu = \overline{X}$ = 58.2, and σ = 16.61). First we calculate the z-score:

$$z = \frac{X - \mu}{\sigma} = \frac{80 - 58.2}{16.61} = 1.31$$

Now, going to a z-score table (see Section 13), we find the area under the curve from 0 to 1.31 is 0.4049. However, we want the value beyond z = 1.31, so we must subtract the z-score from 0.5 (i.e., ½ of 1). Consequently the answer we are looking for is:

$$0.5 - 0.4049 = 0.0951 = 9.51\%$$

In other words, there is a 9.51% chance that we could get a reading of 80 or greater based on our samples (assuming the data follows a normal distribution).

2.11 Chi-Squared

The chi-squared test is used to assess two types of statistical comparison: tests of goodness of fit, and tests of independence.

The chi-square value (determined by the following equation) can be used to determine a p-value by comparing the value of the statistic to a chi-squared distribution table.

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \qquad (59)$$

where

χ^2 = Chi squared test statistic

O_i = an observed frequency

E_i = an expected (or theoretical) frequency

i = count

n = the total number

Problem: We toss a coin 200 times and obtain the following results; 108 heads and 92 tails. Is this a reasonable outcome, or can we suspect the coin somehow favors heads?

Solution: First, we can assume that a fair coin toss should give us on average 100

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heads and 100 tails. But we can also assume that there is some variation due to chance, particularly with a small number of coin tosses. This problem is a good application of a Chi-squared test. We can calculate the chi-squared test statistic for this problem as follows:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} = \frac{(O_H - E_H)^2}{E_H} + \frac{(O_T - E_T)^2}{E_T}$$

$$\chi^2 = \frac{(108-100)^2}{100} + \frac{(92-100)^2}{100} = 1.28$$

From a Chi-squared distribution table (see Section 13), we find for 1 degree of freedom (i.e., 2 classes, head and tails, minus 1) a value of 1.28 falls between 90% and 10%. From this we conclude our coin toss results can be accounted for by chance and the coin toss was fair.

2.12 Spearman Rank Correlation

The Spearman Rank Correlation Coefficient was developed for use with data such as ranks. The score runs between 1 and -1. A coefficient of 1 means a perfect positive correlation and -1 means a perfect negative correlation. A coefficient of 0 indicates no correlation. The formula for the Spearman Rank Correlation Coefficient is:

$$r_s = 1 - \frac{6\sum(D^2)}{N(N^2-1)} \tag{60}$$

where

r_s = Spearman's rank correlation coefficient, nondimensional

6 = a constant (it is always used in the formula)

D = the difference between two corresponding variables

N = the number of data pairs

Problem: Two safety inspectors perform surveys in the same 10 locations within a site. They then independently rank the areas based on the number and type of findings. Based on the independent rankings, how likely is it that these two inspectors would have similar findings at other sites? The ranking (from 1 to 10) for each location

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assessed by the inspectors is shown here:

Location	Inspector 1	Inspector 2
1	8	10
2	4	2
3	1	3
4	5	6
5	7	7
6	10	9
7	2	1
8	3	4
9	9	8
10	6	5

Solution: Spearman's rank correlation coefficient, equation (60), provides an acceptable approximation of the uniformity in the two inspector's findings and is easy to calculate.

Location	Inspector 1	Inspector 2	D	D^2
1	8	10	-2	4
2	4	2	2	4
3	1	3	-2	4
4	5	6	-1	1
5	7	7	0	0
6	10	9	1	1
7	2	1	1	1
8	3	4	-1	1
9	9	8	1	1
10	6	5	1	1
			Sum	18

$$r_s = 1 - \frac{6\sum\left(D^2\right)}{N\left(N^2-1\right)} = 1 - \frac{6(18)}{10\left(10^2-1\right)} = 0.89 = 89\%$$

Therefore, from this analysis, we can conclude there is a strong positive correlation between the two inspectors.

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2.13 Correlation Coefficient

The linear correlation coefficient (usually denoted by the letter r) is a measure of the strength and direction of a linear relationship between two variables (here, x and y). The value of r is a dimensionless quantity such that $-1 \leq r \leq +1$. If x and y have a strong positive linear correlation, r is close to +1 (an r value of exactly +1 indicates a perfect positive fit). Positive values indicate a relationship between x and y variables such that as values for x increase, values for y also increase. If x and y have a strong negative linear correlation, r is close to -1 (an r value of exactly -1 indicates a perfect negative fit). Negative values indicate a relationship between x and y such that as values for x increase, values for y decrease. If there is no linear correlation or a weak linear correlation, *r* is close to 0.

$$r = \frac{N\sum(XY) - \left(\sum X\right)\left(\sum Y\right)}{\sqrt{\left[N\sum\left(X^2\right) - \left(\sum X\right)^2\right]\left[N\sum\left(Y^2\right) - \left(\sum Y\right)^2\right]}} \tag{61}$$

where

X and Y are two variables being evaluated

Equation (61) may be written in an equivalent, but somewhat more simple form:

$$r = \frac{\sum xy}{\sqrt{\left(\sum x^2\right)\left(\sum y^2\right)}} \tag{62}$$

where

$x = X - \overline{X}$

$y = Y - \overline{Y}$

Problem: Calculate the linear correlation coefficient for the following data set.

X	Y
1	2
2	5
3	6

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Solution: We will use equation (62); that requires the average of the x and y values. These are easily found to be 2 and 4.33, respectively.

$x = X - \overline{X}$	$y = Y - \overline{Y}$	xy	x^2	y^2
-1.00	-2.33	2.33	1.00	5.44
0.00	0.67	0.00	0.00	0.44
1.00	1.67	1.67	1.00	2.78
	$\sum$	4.00	2.00	8.67

$$r = \frac{\sum xy}{\sqrt{\left(\sum x^2\right)\left(\sum y^2\right)}} = \frac{4.0}{\sqrt{(2)(8.67)}} = 0.961$$

A linear correlation coefficient of 0.961 indicates a strong positive relationship between the data. Note: Although this sample problem only uses three data pairs, the method is typically used for larger data sets.

2.14 Lower Confidence Limit

With regard to the permissible exposure limit (PEL), the lower confidence limit can be considered the lowest value that the true exposure could be with some degree of confidence (e.g., 95% or 99%). This is written as:

$$LCL = \frac{C_A}{PEL} - \frac{SAE\sqrt{T_1^2C_1^2 + T_2^2C_2^2 + \ldots + T_n^2C_n^2}}{PEL\left(T_1 + T_2 + \ldots + T_n\right)} \tag{63}$$

where

LCL = lower confidence limit, ppm

C_A = time-weighted average concentration of consecutive samples, ppm

PEL = permissible exposure limit, ppm

SAE = sampling and analytical error, see equation (53)

T_n = duration of sample n, minutes

C_n = concentration of sample n, ppm

n = total number of samples

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Problem: Chlorine is used in a process and the following measurements of airborne concentrations are made: 0.75 ppm for 90 min, 0.45 ppm for 170 min, and 0.55 for 220 min. Find the lower confidence limit for this data. Assume the PEL for chlorine is 0.5 ppm and the SAE for this method is 20%.

Solution: First, we need to calculate the time-weighted average of the chlorine samples:

$$C_A = \frac{(0.75\,\text{ppm})(90\,\text{min}) + (0.45\,\text{ppm})(170\,\text{min}) + (0.55\,\text{ppm})(220\,\text{min})}{(90\,\text{min}) + (170\,\text{min}) + (220\,\text{min})} = 0.552\,\text{ppm}$$

Then equation (63) is used to determine to LCL,

$$LCL = \frac{0.552}{0.5} - \frac{0.2\sqrt{0.75^2 90^2 + 0.45^2 170^2 + 0.55^2 220^2}}{0.5(90+170+220)} = 0.97$$

Therefore, since the *LCL* is less than 1.0, we conclude that the exposure does not exceed the PEL at the 95% confidence level.

2.15 Two-Sided 90% Confidence Interval

Given the mean value of a data set, as well as the standard deviation and number of samples in that data set, the two-sided 90% confidence interval is calculated as follows:

$$90\%Conf = \bar{X} \pm 1.645\left(\frac{\sigma}{\sqrt{n}}\right) \tag{64}$$

where

$90\%Conf$ = the two-sided 90% confidence value, units to match $\bar{X}$, σ and n

$\bar{X}$ = arithmetic mean of the sample

σ = standard deviation

n = sample size

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2.16 Two-Sided 95% Confidence Interval

Given the mean value of a data set, as well as the standard deviation and number of samples in that data set, the two-sided 95% confidence interval is calculated as follows:

$$95\%Conf = \bar{X} \pm 1.96\left(\frac{\sigma}{\sqrt{n}}\right) \tag{65}$$

where

$95\%Conf$ = the two-sided 95% confidence value, units to match $\bar{X}$, σ and n

$\bar{X}$ = arithmetic mean of the sample

σ = standard deviation

n = sample size

2.17 One-Sided 95% Confidence Interval

Given the mean value of a data set, as well as the standard deviation and number of samples in that data set, the one-sided 95% confidence interval can be calculated as follows:

$$95\%Conf = \bar{X}\,[+or-]\,1.645\left(\frac{\sigma}{\sqrt{n}}\right) \tag{66}$$

where

$95\%Conf$ = the one-sided 95% confidence value, units to match $\bar{X}$ and σ

$\bar{X}$ = arithmetic mean of the sample

σ = standard deviation

n = sample size

Problem: Several air samples are taken to determine the airborne concentration of a process solvent. The following concentrations (in ppm) are found; 51, 76, 49, 79, 36. Assuming a normal distribution, what is the two-sided 90% confidence interval?

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Solution: The formula for the two-sided 90% confidence interval is given in equation (64). For the data set in the problem, the arithmetic mean and standard deviation were derived in the samples problems above ($\bar{X}$ = 58.2, and σ = 16.61). There were five samples, so n = 5.

$$90\%Conf = \bar{X} \pm 1.645\left(\frac{\sigma}{\sqrt{n}}\right) = 58.2 \pm 1.645\left(\frac{16.61}{\sqrt{5}}\right)$$

$$58.2 \pm 1.645\left(\frac{16.61}{\sqrt{5}}\right) = 58.2 \pm 12.22$$

$$58.2 \pm 12.22 = 45.98, 70.42$$

Therefore, the two-sided 90% confidence interval for the sample set is 45.98 ppm and 70.42 ppm.

Equations (64) and (65) are solved in the same manner; the only difference is the choice of the confidence level desired. However, equation (66) is for a one sided confidence interval, so you must decide if you need the upper or lower confidence interval and use the equation as such (i.e., + or -, not both).

2.18 Permutations and Combinations

Permutations and combinations are mathematical terms applied to the two rules by which items are selected from a group of items. Which rule (equation) to apply is determined by the importance of order in the selection.

2.18.1 Permutation

The number of ways of obtaining an *ordered* subset of k elements from a set of n elements is given by:

$$P_k^n = \frac{n!}{(n-k)!} \tag{67}$$

where

P_k^n = the number of ways of obtaining an *ordered* subset of k elements from a set of n elements

n = total number of items from which to select

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k = number of items taken each time

n! In mathematics, the *factorial* of a positive integer *n*, denoted by *n*!, is the product of all positive integers less than or equal to *n* (e.g., 4! = 4 x 3 x 2 x 1 = 24). Also note that 0! = 1.

2.18.2 Combination

The number of ways of picking *k unordered* outcomes from *n* possibilities is given by:

$$C_k^n = \frac{n!}{k!(n-k)!} \tag{68}$$

where

C_k^n = the number of ways of obtaining an *unordered* subset (combination) of *k* elements from a set of *n* elements

n = total number of items from which to select

k = number of items taken each time

The terms *ordered* and *unordered* can be a bit confusing. The term *unordered* may seem less restrictive, and as a result, more options appear available. This situation is not true. For example, say you have to pick two people from a group of ten, and you pick Al and Beth. In an unordered set, Al and Beth are the same as Beth and Al, so in an unordered set there are actually fewer options.

Problem: At a production facility, there are three standby generators provided so that the back-up electrical power has a high degree of reliability. If only two are required to provide the required capacity, how many combinations of two generators are provided by the set of three? Also, if the three generators are labeled A, B and C, how many ways can they be arranged in a row?

Solution: To answer the first question, we use a combination with n = 3 and k = 2.

$$C_k^n = \frac{n!}{k!(n-k)!} = \frac{3!}{2!(3-2)!} = 3$$

For the second question, we use a permutation because we want to range three out of three, so in this case n=3 and k=3:

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$$P_k^n = \frac{n!}{(n-k)!} = \frac{3!}{(3-3)!} = 6$$

Remember that factorial of zero is one; i.e., 0! =1.

2.19 Poisson Distribution

The Poisson distribution expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate and independently of the time since the last event. It is typically applied to rare events. Mathematically it can be written as:

$$P(r) = \frac{(\lambda t)^r e^{-\lambda t}}{r!} = \frac{(t/m)^r e^{-t/m}}{r!} \tag{69}$$

where

P(r) = probability of *r*, based on a Poisson distribution

λ = expected number of events over time *t*

t = time period

r = number of occurrence of an event

m = *1/λ* = time period per event

e = natural logarithm, 2.71828…

Another way of showing the probability function of the Poisson distribution is:

$$P_m = P\{X = m\} = \frac{a^m e^{-a}}{m!} \tag{70}$$

where

$a = \lambda t$

Problem: Electrical power to a factory fails an average of 5 times every year (e.g., storms, high winds, etc.). Assuming a Poisson distribution, calculate the probability that there will not be more than one failure during a particular week.

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Solution: Since this is a Poisson distribution question, we use equation (70). *Hint: For this you must calculate the probability of no failures in a week and the probability of one failure in a week and sum them.* Fist we calculated the average failure rate:

$$a = \lambda t = \left(\frac{5\,\text{failures}}{52\,\text{week}} \right) (1\,\text{week}) = 0.096$$

$$P(r) = \frac{a^m e^{-a}}{m!} = \frac{(0.096)^0 e^{-0.096}}{0!} + \frac{(0.096)^1 e^{-0.096}}{1!} = 0.996$$

Remember: Any number raised to the zero power, 0! and 1! all equal 1.

2.20 Reliability

In simple terms, reliability is defined as the probability that a device will perform its required function for a specific period of time (i.e., reliability is the probability of no failure). Mathematically this probability can be defined as:

$$R(t) = e^{-\lambda t} \tag{71}$$

where

R(t) = reliability as a function of time, $0 \le R(t) \le 1$

λ = the failure rate (also called the hazard rate) which predicts the number of failures that have occurred over a period of time

t = time

Problem: Electrical power to a factory fails an average of 5 times every year (e.g., storms, high winds, etc.). Calculate the reliability of the power system over a one-week period.

Solution:

$$R(t) = e^{-\lambda t} = e^{-\left(\frac{5\,\text{failures}}{52\,\text{weeks}}\right) 1\,\text{week}} = 0.908$$

Based on this calculation, the power supply system has a reliability of about 91%.

The probability of failure is simply the complement of the reliability probability, and can be written as:

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$$P_f + P_s = 1 \qquad (72)$$

where

P_f = probability of failure

P_s = probability of success, which is equal to $R(t)$

From these equations we can write:

$$P_f = 1 - R(t) \qquad (73)$$

and

$$P_f = 1 - P_s \qquad (74)$$

Problem: Based on the reliability just calculated, what is the failure probability of the electrical supply system over a one-week period?

Solution: For this calculation, use equation (73) and use the reliability rate just calculated.

$$P_f = 1 - R(t) = 1 - 0.908 = 0.092$$

Based on this calculation, there is about a 9% probability of electrical system failure in a week.

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Engineering Economics

3 Engineering Economics

Engineering economics, previously known as engineering economy, is a subset of economics that is concerned with the application of economic techniques to the evaluation of design and engineering alternatives. Engineers should seek solutions to problems that are technically sound but in which the economic viability of each potential solution is also considered. However, the following equations are not special "engineering" equations; they apply universally to financial projections.

The first equation calculates the future value of a lump sum payment made today given an interest rate compounding over a number of years.

$$F = P\left(1+i\right)^{n} \tag{75}$$

where

F = future value of money, \$

P = present value of money, \$

i = interest rate, percent in decimal form

n = number of years

Equation (75) can be re-arranged to calculate P given F (same units).

$$P = F\left(1+i\right)^{-n} \tag{76}$$

Problem: Personal protective equipment has a current replacement cost of \$15,000. Assuming an inflation increase of 3% per year, what will be the adjusted cost of the PPE in 5 years when it is expected to be replaced?

Given the expected replacement cost and assuming the cost allocated for the PPE replacement can be put into an interest bearing account that yields 5% per year, how much should be invested today to cover the PPE costs in 5 years?

Solution: The two problems can be solved with equations (75) and (76), respectively.

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First, the cost of the PPE in 5 years is:

$$F = P\left(1+i\right)^{n} = \$15,000\left(1+0.03\right)^{5} = \$17,389.11$$

Next, based on this amount ($17,389.11) and assuming we earn 5% interest, the amount we would need to invest today is:

$$P = F\left(1+i\right)^{-n} = \$17,389.11(1+0.05)^{-5} = \$13,628.82$$

The following equation can be used to calculate the future value of a series of annual payments given an interest rate and number of years.

$$F = A\left[\frac{\left(1+i\right)^{n}-1}{i}\right] \tag{77}$$

where

F = future value of money, $

A = annual payment, $

i = interest rate, percent in decimal form

n = number of years

Equation (77) can be re-arranged to calculate A given F (same units).

$$A = F\left[\frac{i}{\left(1+i\right)^{n}-1}\right] \tag{78}$$

Problem: Continuing with the PPE replacement problem above; calculate how much would you have to place into the account each year for 5 years instead of one lump sum today.

Also, you evaluate your PPE budget and it appears you can place $3500 into the same account each year over the next 5 years. How much will be available for PPE purchase in 5 years?

Solution: First, use equation (78) to find the answer to the first question:

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$$A = F\left[\frac{i}{(1+i)^n - 1}\right] = \$17,389.11\left[\frac{0.05}{(1+0.05)^5 - 1}\right] = \$3,146.99$$

Next, use equation (77) to see how much would accumulate based on the yearly contributions to the interest-bearing account:

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = \$3,500\left[\frac{(1+0.05)^5 - 1}{0.05}\right] = \$19,339.71$$

The following equation is used to calculate the present value of a series of equal annual payments given an interest rate and number of years.

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \tag{79}$$

where

P = present value of money, \$

A = annual payment, \$

i = interest rate, percent in decimal form

n = number of years

Equation (79) can be re-arranged to calculate A given P (same units); it is known as capital recovery.

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \tag{80}$$

Problem: Your company is considering the purchase of a new lab analyzer. One model (Model A) costs \$4500. A second model (Model B) costs more, \$6000, but requires \$375 less each year in replacement parts and supplies. Using the concept of present worth, evaluate which option is more cost effective over a 7 year period (the expected service life of both). Assume an interest rate of 4%.

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Solution: First, we must calculate the present worth of the $375 saved each year over the 7 year period. For this we use equation (79):

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = \$375\left[\frac{(1+0.04)^7 - 1}{0.04(1+0.04)^7}\right] = \$2250.77$$

Next, we need to subtract this amount from the present value of Model B since it would be "paying" back this amount each year:

$$\$6,000 - \$2250.77 = \$3749.23$$

This is less than the present value of Model A, $4500, so Model B is the more cost-effective.

Problem: A testing lab is considering the addition of a new gas chromatograph that has a purchase price of about $18,000. Ignoring other costs, estimate the yearly cost that should be charged to clients to offset the acquisition. Assume an interest rate of 3.5% and a service life of 6 years with negligible salvage value.

Solution: For this we use equation (80):

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = \$18,000\left[\frac{0.035(1+0.035)^6}{(1+0.035)^6 - 1}\right] = \$3378.02$$

In other words, if $3378.02 is charged each year for the use of the GC, the cost of the GC will be recouped in 6 years, assuming an interest rate of 3.5%.

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Chemistry and Concentrations

4 Chemistry and Concentrations

4.1 Ideal Gas Law

The ideal gas law (also called the perfect gas law) is the equation of state of a hypothetical ideal gas. It provides a good approximation of the behavior of many gases under many conditions, such as air and other gases typically encountered in industrial hygiene and safety applications. The ideal gas law can be written as:

$$P \cdot Vol = n \cdot R \cdot T \tag{81}$$

where

P = absolute pressure of the gas, atm

Vol = volume of gas, liters (l)

n = amount of gas, gram moles

R = gas constant, 0.082 l-atm/gram moles-K

T = temperature, K

These are the typical units used in safety and industrial hygiene applications. However, other applications (or simply personal preference) may employ different units. See the following table.

Ideal Gas Law Gas Constant (R)							
			Absolute Pressure				
Volume	**Temp**	**moles**	**atm**	**psi**	**mm Hg**	**in Hg**	**ft H_2O**
ft^3	**K**	gm	0.00290	0.0426	2.20	0.0867	0.0982
		lb	1.31	19.31	999.0	39.3	44.6
	°R	gm	0.00161	0.02366	1.22	0.0482	0.0546
		lb	0.730	10.73	555.0	21.85	24.8
liters	**K**	gm	0.08206	1.206	62.4	2.45	2.78
		lb	37.2	547.0	28300.0	1113.0	1262.0
	°R	gm	0.0456	0.670	34.6	1.36	1.55
		lb	20.7	304.0	15715.0	619.0	701.0

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For a gas at two varying conditions, equation (81) can be written as:

$$\frac{P_1Vol_1}{nRT_1} = \frac{P_2Vol_2}{nRT_2} \tag{82}$$

With n and R constant, equation (82) can be written:

$$\frac{P_1Vol_1}{T_1} = \frac{P_2Vol_2}{T_2} \tag{83}$$

Problem: Propane has a chemical composition of C_3H_8 yielding a molecular weight of 44. Calculate its density in lbs/ft^3 at 1 atmosphere and 68 ^{o}F.

Solution: First, we can take equation (81) and multiply each side of the equation by the molecular weight (MW); to provide:

$$MW \cdot P \cdot Vol = MW \cdot n \cdot R \cdot T$$

This can be rearranged to:

$$MW \cdot P = \left(\frac{MW \cdot n}{Vol}\right) R \cdot T$$

The term $\left(\frac{MW \cdot n}{Vol}\right)$ is the density (ρ), so we can write:

$$MW \cdot P = \rho \cdot R \cdot T$$

which can be rearranged to solve for d:

$$\rho = \frac{MW \cdot P}{R \cdot T}$$

Selecting the appropriate value for R (based on the units we desire) we find:

$$\rho = \frac{44 \cdot 1\,\text{atm}}{\left(0.73\,\text{ft}^3 \cdot \text{atm/lb mole} \cdot \text{R}\right) \cdot \left(460 + 68\text{F}\right)} = 0.114\,\text{lbs/ft}^3$$

Problem: A small oxygen cylinder is full, and at room temperature the gauge reads 1500 psi. The cylinder is left in an area where the ambient temperature can climb as high as 90 ^{o}F. What pressure would the gauge read at that temperature? Assume room temperature is 70 ^{o}F.

Solution: We can use equation (83), and since the volume of the cylinder does not change, the equation can be written as:

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$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Solving for P_2 and remembering to use degrees Rankine leads to:

$$P_2 = \frac{P_1}{T_1} T_2 = \frac{1500\text{psi}}{(70\text{F} + 460\text{F})}(90\text{F} + 460\text{F}) = 1556.6\,\text{psi}$$

The pressure increase is not that substantial in this case.

4.2 Concentration of Vapors and Gases

The calculation of concentrations of airborne contaminants is a common effort in safety and industrial hygiene. Equations related to permissible exposure limits are also commonly encountered. These are presented in the following section.

4.3 Airborne Concentration via Volume

The following expression may be used to calculate the airborne concentration of a gas or vapor in part-per-million (ppm) based on volume.

$$ppm = \frac{V_{contam}}{V_{air}} x10^6 \tag{84}$$

where

ppm = airborne concentration, ppm

V_{contam} = volume of contaminant (units to match V_{air})

V_{air} = volume of air (units to match V_{contam})

10^6 = conversion factor for ppm

Problem: One pound of acetylene leaks from a cylinder into a room that measures 30 ft wide by 50 ft long by 12 feet high. Assume acetylene as a density of 0.0682 lbs/ft^3 at room temperature and pressure. What is the concentration in ppm (assume uniform mixing and no losses)?

Solution: First, if we take the inverse of the density, we find acetylene occupies 14.66 ft^3/lb, so we write:

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$$ppm = \frac{V_{contam}}{V_{air}} x10^6 = \frac{14.66\,\text{ft}^3}{(30\,\text{ft})(50\,\text{ft})(12\,\text{ft})} x10^6 = 814.4\,\text{ppm}$$

4.4 Dalton's Law of Partial Pressure (Gas) & Raoult's Law (Liquids)

Dalton's law (also called Dalton's law of partial pressures) states that the total pressure exerted by a gaseous mixture is equal to the sum of the partial pressures of each individual component in a gas mixture. It can be written as:

$$P_{total} = X_1 P_1 + X_2 P_2 + \ldots + X_i P_i \tag{85}$$

where

P_{total} = total pressure of gas mixture, mmHg

X_i = mole fraction of gas i in the mixture, non dimensional

P_i = pressure of gas i in the mixture, mmHg

Note that the partial pressure of each component is:

$$P_{partial-i} = X_i P_i \tag{86}$$

Raoult's law states the vapor pressure of an ideal solution is dependent on the vapor pressure of each chemical component and the mole fraction of the component present in the solution. Mathematically, Raoult's law can be written the same as Dalton's law, but applied to problems involving solutions.

Problem: An air compressor supplies air at 400 psi. Assuming air is comprised of oxygen (21%) and nitrogen (79%), what is the partial pressure of the oxygen and nitrogen?

Solution: Since we know the total pressure and percent fractions, the solution is found by multiplying the oxygen (21%) and nitrogen (79%) fractions by the total pressure to arrive at the partial pressures contributed by each:

$$P_{nitrogen} = (0.79)(400\,\text{psi}) = 316\,\text{psi}$$
$$P_{oxygen} = (0.21)(400\,\text{psi}) = 84\,\text{psi}$$

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4.5 Airborne Concentration via Pressure

The following expression may be used to calculate the airborne concentration of a gas or vapor in part-per-million (ppm) based on pressure.

$$ppm = \frac{P_v}{P_{atm}} x 10^6 \tag{87}$$

where

ppm = airborne concentration, ppm

P_v = vapor pressure of contaminant (units to match P_{atm})

P_{atm} = vapor pressure of air (units to match P_v)

10^6 = conversion factor for ppm

Problem: Isopropyl alcohol (IPA) has a vapor pressure of 44 mmHg at 25 °C. What is the equilibrium concentration (in ppm) in air around the IPA source assuming a temperature of 25 °C and a pressure of 1 atmosphere?

Solution: Knowing 1 atmosphere equals 760 mmHg, we can write:

$$ppm = \frac{P_v}{P_{atm}} x 10^6 = \frac{44\,\text{mmHg}}{760\,\text{mmHg}} x 10^6 = 57,895\,\text{ppm}$$

4.6 Conversion for Airborne Concentrations: ppm (to/from) mg/m³

In addition to quantifying airborne contaminants in units of ppm, another common set of units is mg/m^3. The following equation can be used to make this conversion.

$$ppm = \frac{mg/m^3 \; x \; 24.45}{MW} \tag{88}$$

where

ppm = airborne concentration, ppm

mg/m^3 = airborne concentration, mg/m^3

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24.45 = molar volume of any gas or vapor at STP, l/gram mole

MW = molecular weight of contaminant, g/gram mole

Problem: Isopropyl alcohol (IPA) has a chemical formula of C_3H_8O, and therefore a molecular weight of 60. Using the equilibrium concentration just calculated above (57,895 ppm), calculate the equilibrium concentration in mg/m^3 of the IPA in air.

Solution: We need to re-arrange equation (88) as follows:

$$mg / m^3 = \frac{(ppm)(MW)}{24.45} = \frac{(57,895)(60)}{24.45} = 142,073\,\text{mg/m}^3 = 142\,\text{kg/m}^3$$

4.7 Conversion for Airborne Concentrations: ppm (to/from) g/l

Another volumetric conversion, this for converting between ppm and grams-per-liter (g/l) is:

$$C = \frac{g \cdot 24.45x10^6}{MW \cdot V} \tag{89}$$

where

C = airborne concentration, ppm

g = airborne concentration, grams

24.45 = molar volume of any gas or vapor at STP, l/mole

MW = molecular weight of contaminant, g/mole

V = Volume, liters (l)

Problem: A carbon dioxide (CO_2) test gas is prepared by placing 1 gram of CO_2 into a 10 liter container. What is the concentration (ppm) of the CO_2-air mixture?

Solution: The molecular weight of CO_2 is 44 and the other required data are provided in the question, so we can use equation (89) to find the solution:

$$C = \frac{g \cdot 24.45x10^6}{MW \cdot V} = \frac{1\,\text{g} \cdot 24.45x10^6}{44 \cdot 10\,\text{liter}} = 55,568\,\text{ppm}$$

This is well over the published IDLH value of 40,000 ppm.

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4.8 Threshold Limit Value (TLV) of Airborne Mixture

The Threshold Limit Value (TLV) of a single airborne contaminant can be found by looking it up in a table of permitted exposure limits. But what if more than one contaminant is present; how is the TLV of the mixture determined? To do this, the following expression is used. If the resulting TLV_{mix} is equal or greater than 1.0, the mixture exceeds the TLV.

$$TLV_{mix} = \frac{C_1}{TLV_1} + \frac{C_2}{TLV_2} + \ldots + \frac{C_n}{TLV_n} \quad (90)$$

where

TLV_{mix} = TLV ratio of the airborne mixture, nondimmensional

C_n = measured airborne concentration of contaminant n

TLV_n = permitted airborne concentration of contaminant n

Problem: Air samples find toluene concentrations at 35 ppm and benzene concentrations at 0.25 ppm within the same air sample. If the TLVs are 50 ppm and 0.5 ppm, respectively, is the combined TLV exceeded?

Solution: Substituting directly into equation (90) yields:

$$TLV_{mix} = \frac{C_1}{TLV_1} + \frac{C_2}{TLV_2} = \frac{35\,\text{ppm}}{50\,\text{ppm}} + \frac{0.25\,\text{ppm}}{0.5\,\text{ppm}} = 1.2$$

Therefore the combined TLV of the mixture is exceeded.

4.9 Threshold Limit Value (TLV) of Liquids

For liquid mixtures, a similar approach is taken, except the actual TLV of the mixture is calculated as follows:

$$TLV_{mix} = \frac{1}{\dfrac{F_1}{TLV_1} + \dfrac{F_2}{TLV_2} + \ldots + \dfrac{F_n}{TLV_n}} \quad (91)$$

where

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TLV_{mix} = TLV of the liquid mixture, mg/m^3

F_n = weight fraction of chemical n, decimal percent

TLV_n = TLV of chemical n, mg/m^3

Problem: What is the TLV of a 50/50 mixture of hexane and xylene? Assume the TLV for hexane is 176 mg/m^3 and the TLV for xylene is 434 mg/m^3.

Solution: Substituting into equation (91) yields:

$$TLV_{mix} = \frac{1}{\frac{F_1}{TLV_1} + \frac{F_2}{TLV_2}} = \frac{1}{\frac{.50}{176\,\text{mg/m}^3} + \frac{.50}{434\,\text{mg/m}^3}} = 250\,\text{mg/m}^3$$

4.10 Le Chatelier's Rule

The estimated lower flammability limit (LFL) of a mixture of combustible gases can be calculated using Le Chatelier's Rule:

$$LFL_{mix} = \frac{1}{\frac{f_1}{LFL_1} + \frac{f_2}{LFL_2} + \ldots + \frac{f_n}{LFL_n}} \tag{92}$$

where

LFL_{mix} = LFL of the gas mixture, %

f_n = volume fraction of flammable gas n, decimal form

LFL_n = LFL of flammable gas n, %

Note: Although this formula calculates LFL, the same approach can be used for the upper flammability limit (UFL).

Problem: What is the LFL of the following mixture: methane (75%), ethane (15%) and propane (10%)?

Solution: Consulting MSDS of other suitable sources, we find the following LFLs: methane (5%), ethane (3%) and propane (2.1%). We can substitute the above fractions and LFLs into equation (92) to find:

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$$LFL_{mix} = \frac{1}{\frac{f_1}{LFL_1} + \frac{f_2}{LFL_2} + \ldots + \frac{f_n}{LFL_n}} = \frac{1}{\left(\frac{.75}{5}\right) + \left(\frac{.15}{3}\right) + \left(\frac{.10}{2.1}\right)} = 4.0\,\%$$

4.11 Vapor-Hazard Ratio

The vapor-hazard ratio is a simple ratio of the saturation concentration of an airborne contaminant to permitted concentration. Since it is a ratio of the two values, it indicates a relative level of risk that includes the volatility of the contaminant. The vapor-hazard ratio is expressed as:

$$vapor\text{ - }hazard\ ratio = \frac{sat.\,concentration}{exposure\ guideline} \tag{93}$$

where

vapor-hazard ratio = relative level of risk of an airborne contaminant, non dimensional

sat. concentration = saturation concentration of gas (or vapor), ppm

exposure guideline = concentration permitted by guidelines, ppm

Problem: What is the Vapor-Hazard Ratio of methyl ethyl ketone (MEK)?

Solution: From an MSDS for MEK, we find an exposure limit of 200 ppm and a vapor pressure of 78 mmHg (at 20 ^{o}C). Standard atmospheric pressure is 760 mmHg. We can then use equation (87) to find the saturation pressure:

$$ppm = \frac{P_v}{P_{atm}} x10^6 = \frac{78\,\text{mmHg}}{760\,\text{mmHg}} x10^6 = 102{,}632\,\text{ppm}$$

Equation (93) can then be used to find the Vapor-Hazard Ratio:

$$vapor\ /\ hazard\ ratio = \frac{102632\ ppm}{200ppm} = 513$$

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4.12 Reduction Factor – Day

Many occupational limits for exposure are based on an 8 hour work-day, and a 40 hour work-week. When an employee works an altered schedule, a Reduction Factor for an unusual work schedule can be used to adjust exposure limits based on the actual hours worked in a day. For a modified work day, this can be written as:

$$RF_{day} = \frac{8}{h} x \frac{24-h}{16} \tag{94}$$

where

RF_{day} = reduction factor, nondimensional

h = number of hours worked in a day

4.13 Reduction Factor – Week

Similar to above, when an employee works an altered work week, a Reduction Factor can be used to adjust exposure limits based on the actual hours worked in a week. For a modified work week, this can be written as:

$$RF_{week} = \frac{40}{h_w} x \frac{168-h_w}{128} \tag{95}$$

where

RF_{week} = reduction factor, nondimensional

h_w = number of hours worked in a week

Problem: A worker is exposed to toluene during his shift. The TLV for toluene is 50 ppm. If the worker works 9 hours in a day, what is the permitted exposure to toluene? If the worker works 9 hours per day all week (5 days) what is the permitted exposure?

Solution: First we can calculate the reduction factors for one day and one week based on the hours worked:

$$RF_{day} = \frac{8}{h} x \frac{24-h}{16} = \frac{8}{9} x \frac{24-9}{16} = 0.83$$

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$$RF_{week} = \frac{40}{h_w} x \frac{168 - h_w}{128} = \frac{40}{50} x \frac{168 - 50}{128} = 0.74$$

Therefore, the permitted exposure for an increased day and week is:

$$TLV_{permitted-day} = 0.83(50\,\text{ppm}) = 41\,\text{ppm}$$

$$TLV_{permitted-week} = 0.74(50\,\text{ppm}) = 37\,\text{ppm}$$

Notice the week value is not the same as the day value, even though it is based on the same increase in hours per work day.

4.14 Chemistry of Solutions

4.14.1 Beer-Lambert Law (Beer's Law)

One form of Beer's law can be used to evaluate the presence of a contaminant in a solution based on the amount of light absorbed by the solution. This is written as:

$$\log \frac{I_o}{I} = A = abc \tag{96}$$

where

A = absorbance, nondimensional

I_o = intensity of incident light

I = intensity of transmitted (exiting) light

a = molar absorptivity constant, L/g-cm

b = length of light beam path, cm

c = concentration of absorbing material, g/L

Note that the units for I_o and I are not specified above. Since they are expressed as a fraction, the units only need to be consistent.

Problem: A solution reduces the amount of light transmitted through it to 1/5 the original intensity. If the molar absorptivity has been found to be 2.04 L/g-cm and the beam length is 1.2 cm, what is the concentration of the solution?

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Solution: First, we can use equation (96) to solve for the absorbance:

$$\log\frac{I_o}{I} = A = \log\frac{5}{1} = 0.7$$

Equation (96) can then be re-arranged to solve for the concentration:

$$c = \frac{A}{ab} = \frac{0.7}{(2.04\,\text{L/g-cm})(1.2\,\text{cm})} = 0.286\,\text{g/L}$$

4.14.2 pH Calculation

The pH of a solution indicates if the solution is an acid, base, or neutral. Therefore, pH can indicate potential hazards of solutions. pH is a measure of the hydrogen ions in solution and pH is calculated as follows:

$$pH = -\log_{10}\left[H^+\right] \tag{97}$$

where

pH = a quantitative description of acidity or alkalinity of a solution (ranges from 0-14)

$\left[H^+\right]$ = hydrogen ion concentration, gram moles/liter (= Molarity, = M)

Problem: Calculate the pH of a solution that has 5.0 grams of HNO_3 in 2.0 liters of solution. The molecular weight of HNO_3 is 63.01 g/mole.

Solution: First we need to calculate the number of moles of HNO_3:

$$\frac{5.0\,\text{grams}}{63.01\,\text{grams/mole}} = 0.0794\,\text{moles}$$

Then we can calculate the molarity of the solution:

$$M = \frac{0.0794\,\text{moles}}{2.0\,\text{liters}} = 0.0397M$$

Finally, we can use equation (97) to find the pH:

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$$pH = -\log_{10}\left[H^+\right] = -\log_{10}\left[0.0397\right] = 1.4$$

4.14.3 Acid Dissociation Constant

In simple terms, the acid dissociation constant, K_a, is a quantitative measure of the strength of an acid in solution. It is calculated as follows:

$$K_a = \frac{\left[H^+\right] x \left[A^-\right]}{\left[HA\right]} \tag{98}$$

where

K_a = acid dissociation constant, nondimensional

$\left[H^+\right]$ = hydrogen ion concentration, M

$\left[A^-\right]$ = concentration of conjugate base of a weak acid, M

$\left[HA\right]$ = weak acid concentration, M

Problem: A solution of acetic acid ($C_2H_4O_2$) in water has a pH of 2.54 and a molarity of 0.462. What is the acid dissociation constant, K_a?

Solution: First, we can use equation (97) to find the hydrogen ion concentration of the solution.

$$pH = -\log_{10}\left[H^+\right]$$

Which can be re-arranged to solve for the hydrogen ion concentration,

$$\left[H^+\right] = 10^{-pH} = 10^{-2.54} = 0.002884\,\mathrm{M}$$

Next, since the ratio of moles of $C_2H_3O_2^-$ to H^+ is 1:1; we can use equation (98) to write:

$$K_a = \frac{\left[H^+\right] x \left[A^-\right]}{\left[HA\right]} = \frac{\left[0.002884\,\mathrm{M}\right] x \left[0.002884\,\mathrm{M}\right]}{0.462\,\mathrm{M}} = 1.8\,\mathrm{x}\,10^{-5}$$

This value can be compared to those published for K_a.

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4.14.4 Base Dissociation Constant

In simple terms, the base dissociation constant, K_b, is a quantitative measure of the strength of a base in solution. It is calculated as follows:

$$K_b = \frac{\left[BH^+\right] x \left[OH^-\right]}{\left[B\right]} \tag{99}$$

where

K_b = base dissociation constant, nondimensional

$\left[BH^+\right]$ = concentration of positive ions from ionized base, M

$\left[OH^-\right]$ = hydroxide ion concentration, M

$\left[B\right]$ = concentration of non-ionized base, M

Problem: What is the pH of a 0.10 M solution of methylamine (CH_5N) in water? Note: Methylamine has a base dissociation constant of $4.4x10^{-4}$.

Solution: First, we can write the chemical equation as:

$$\mathrm{CH_5N + H_20 \Leftrightarrow CH_6N^+ + OH^-}$$

Then we can use equation (99) to write:

$$4.4\,\mathrm{x}\,10^{-4} = \frac{\left[\mathrm{CH_6N^+}\right] x \left[OH^-\right]}{\left[0.10\right]}$$

Since CH_6N^+ and OH^- have a ratio of 1:1 (i.e., equal molarity); we can find they both are:

$$\left[\mathrm{CH_6N^+}\right] = \left[OH^-\right] = \sqrt{\left(0.10\,\mathrm{M}\right)\left(4.4\,\mathrm{x}\,10^{-4}\right)} = 6.63\,\mathrm{x}\,10^{-3}$$

Similar to finding the pH, we can find pOH :

$$pOH = -\log_{10}\left[6.63\,\mathrm{x}\,10^{-3}\right] = 2.18$$

We want to determine pH, so we need to subtract the pOH from 14;

$$pH = 14 - pOH = 14 - 2.18 = 11.82$$

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Thus the pH of the methylamine solution is 11.82.

4.15 Asbestos (Airborne Contaminant)

Various methods are used to assess asbestos concentrations in air. The following presents some of the equations used by those methods.

4.15.1 Asbestos Fiber Concentration by PCM

Airborne asbestos fiber concentrations can be assessed using phase contrast microscopy (PCM). The following equation is used in the analysis:

$$C_{asb} = \frac{(C_s - C_b)A_c}{1000 A_f V_s} \tag{100}$$

where

C_{asb} = airborne concentration of asbestos fibers, fibers/ml

C_s = average number of fibers counted per graticule field in the sample

C_b = average number of fibers counted per graticule field in the field blank

A_c = effective collection area of filter, 385 mm^2 for 25 mm filter

A_f = graticule field area, 0.00785 mm^2

V_s = air volume sample, liters (l)

Problem: What is the airborne concentration of asbestos fibers if 800 liters of air are sampled and 100 fibers are counted on 46 fields, and the field blank has no fibers. Assume the effective area of the filter is 385 mm^2 (25 mm filter) and the graticule field area is 0.00785 mm^2.

Solution: Applying equation (100) and substituting leads to:

$$C_{asb} = \frac{(C_s - C_b)A_c}{1000 A_f V_s} = \frac{(2.17 - 0\,\text{fibers})(385\,\text{mm}^2)}{1000(0.00785\,\text{mm}^2)(800\,\text{L})} = 0.133\,\text{f/mL}$$

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4.15.2 Asbestos Fiber Concentration

Another form of the equation for assessing airborne asbestos fiber concentration is:

$$C_{asb} = \frac{EA_c}{1000V_s} \tag{101}$$

where

C_{asb} = airborne concentration of asbestos fibers, fibers/ml

E = fiber density on filter, fibers/ mm^2 (see next equation)

A_c = effective collection area of filter, 385 mm^2 for 25 mm filter

V_s = air volume sample, liters (l)

Problem: What is the airborne concentration of asbestos fibers if 800 liters of air are sampled and the fiber density is 102 f/mm^2? Assume the effective area of the filter is 385 mm^2 (25 mm filter).

Solution: Applying equation (101) and substituting leads to:

$$C_{asb} = \frac{EA_c}{1000V_s} = \frac{\left(102\,\text{f/mm}^2\right)\left(385\,\text{mm}^2\right)}{1000 \cdot 800\,\text{L}} = 0.049\,\text{fibers/mL}$$

4.15.3 Fiber Density

The fiber density can be calculated as follows:

$$E = \frac{\frac{F}{N_f} - \frac{B}{N_b}}{A_f} \tag{102}$$

where

E = fiber density on filter, fibers/ mm^2

F/N_f = average fiber count per graticule field

B/N_b = average fiber count per graticule field for the field blank

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A_f = graticule field area, 0.00785 mm^2

Problem: What is the fiber density on a filter if 100 fibers are counted on 46 fields, and the field blank has no fibers? Assume the graticule field area is 0.00785 mm^2.

Solution: Using equation (102) and substituting leads to:

$$E = \frac{\frac{F}{N_f} - \frac{B}{N_b}}{A_f} = \frac{\left(\frac{100}{46} - 0\right)\text{fibers}}{0.00785\,\text{mm}^2} = 277\ \text{f/mm}^2$$

4.15.4 Microscopic Limit of Resolution (Abbe's Equation)

The Abbe's equation can be used to determine the limit of resolution for a microscope, which may be required when conducting asbestos sample assessment. The equation can be written as:

$$d = \frac{0.61\lambda}{\eta \sin \alpha} \tag{103}$$

where

d = limit of resolution, nm

0.61 = a constant

λ = wavelength of light used in microscope, nm

η = index of refraction of medium between point source and lens, relative to free space

α = half the angle of the cone of light from specimen plane accepted by the objective, radians

Note: Values for η typically range between 1.0 (air) to about 1.5 (oils). Also, the value ηsinα is often expressed as NA (numerical aperture). Also, radians are related to degrees in the following manner:

$$1 \text{ radian} = \frac{180^o}{\pi} \tag{104}$$

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Problem: Visible light in air is used for a microscope. The wavelength for visible light is 500 nm. What is the limit of resolution for this setup? Assume a half-angle of 40 degrees (0.698 radians) and η = 1.0 for air.

Solution: Using equation (103) and substituting leads to:

$$d = \frac{0.61\lambda}{\eta \sin\alpha} = \frac{(0.61)(500\,\mathrm{nm})}{1.0 \cdot \sin(0.698)} = 475\,\mathrm{nm} = 475\,\mu\mathrm{m}$$

4.16 Particle Settling Velocity

The terminal settling velocity of a spherical particle in a fluid (e.g., air) can be described by:

$$V_{TS} = \frac{g d_p^2 (\rho_p - \rho_a)}{18\eta} \tag{105}$$

where

V_{TS} = terminal settling velocity of particle, cm/sec

g = acceleration due to gravity, cm/sec^2

d = diameter of particle, cm

ρ_p = density of particle, g/cm^3

ρ_a = density of fluid (e.g., air), g/cm^3

η = viscosity of fluid (e.g., air), poise (P)

Note that equation (105) is applicable for particles less than 80 micrometers (µm) in size (i.e., aerodynamic diameter) and having a Reynolds number less than 2.0. Reynolds numbers are presented next.

Problem: A high pressure water spray system generates particles with an average diameter of 80 µm. Calculate the terminal settling velocity of the water particles in still air. Assume the density of water is 1.0 g/cm^3. Also, the density of air is 0.0012 g/cm^3 and its viscosity is 0.000182 Poise. The gravitational acceleration is 980 cm/sec^2.

Solution: Substituting the given data into equation (105) leads to:

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$$V_{TS} = \frac{gd_p^2(\rho_p - \rho_a)}{18\eta} = \frac{\left(980\,\text{cm/sec}^2\right)\left(0.008\,\text{cm}\right)^2(1-0.0012\,\text{g/cm}^3)}{18\left(0.000182\,\text{g/cm-sec}\right)} = 19.12\,\text{cm/sec}$$

4.16.1 Reynolds Number

The Reynolds number expresses the ratio of inertial (resistance to change or motion) forces to viscous (heavy and gluey) forces. The Reynolds number is nondimensional and is used in numerous fluid mechanics applications; it can be calculated using the following equation.

$$R_e = \frac{\rho d v}{\eta} \tag{106}$$

where

Re = Reynolds number, nondimensional

ρ_a = density of fluid (e.g., air), g/cm^3

d = characteristic dimension (here it is the diameter of particle), cm

v = velocity of particle, cm/sec

η = viscosity of fluid (e.g., air), poise (P)

Problem: Calculate the Reynolds number for the particle described in the previous sample problem and determine if the use of equation (105) is appropriate based on the calculated settling velocity.

Solution: Given the data from the previous sample problem, including the calculated settling velocity of 19.12 cm/sec, we can use equation (106) to calculate the Reynolds number.

$$R_e = \frac{\rho d v}{\eta} = \frac{\left(1\,\text{g/cm}^3\right)\left(0.008\,\text{cm}\right)\left(19.12\,\text{cm/sec}\right)}{0.000182\,\text{g/cm-sec}} = 1.01$$

Since the calculated Reynolds number is less than 2.0, equation (105) provides a reasonable approximation of the particle settling velocity.

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Mechanics

5 Mechanics

5.1 Newton's Second Law

Newton's Second Law explains how the velocity of an object changes when it is subjected to an external force. The law defines a force to be equal to change in momentum (mass times velocity) per change in time. Since a change in velocity with respect to time is acceleration, Newton's Second Law can be written as:

$$F = ma \tag{107}$$

where

F = force, lbs

m = mass, slugs

a = acceleration, ft/sec^2

Slug The slug is a unit of mass in the English foot-pound-second system. One slug is the mass accelerated at 1 foot per second per second by a force of 1 pound. Since the acceleration of gravity (*g*) in English units is 32.17 feet per second per second, the slug is equal to 32.2 pounds (14.6 kilograms).

Problem: A roller coaster accelerates from 0 to 50 mph in 5 seconds. What is the force a 100 lb child exerts on the back of her seat?

Solution: First, we must convert the weight in pounds to Slugs:

$$\frac{100\,\text{lbs}}{32.2\,\text{ft/sec}^2} = 3.1\,\text{slugs}$$

We must also calculate the acceleration:

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$$a = \frac{v}{t} = \frac{(50\,\text{mile/hr})(5280\,\text{ft/mile})(\text{hr}\,/\,3600\,\text{sec})}{5\,\text{sec}} = 14.67\,\text{ft/sec}^2$$

Now, using equation (107) we find:

$$F = ma = (3.1\,\text{slugs})(14.67\,\text{ft/sec}^2) = 45.47\,\text{lbs}$$

Therefore, the child experiences about a one-half "g" force during the acceleration.

5.2 Weight

Weight is the force exerted on an object with a given mass due to gravitational acceleration. This is an application of Newton's Second Law and can be written:

$$W = mg \tag{108}$$

where

W = weight, lb-force

m = mass, slugs

g = acceleration due to gravity, ft/sec^2

Problem: An adult weighs 185 pounds; what is his mass?

Solution: Re-arranging equation (108), we find:

$$m = \frac{W}{g} = \frac{185\,\text{lbs}}{32.2\,\text{ft/sec}^2} = 5.75\,\text{slugs}$$

5.3 Momentum

In mechanics, momentum is the product of the mass and velocity of an object,

$$p = mv \tag{109}$$

where

p = momentum, lb/sec

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m = mass, slugs

v =velocity, ft/sec

Problem: A truck weighing 11,000 lbs and traveling at 60 mph strikes the rear of a car that weighs 7,000 lbs and is traveling at 40 mph in the same direction. Assuming that immediately after the crash the vehicles' damage causes them to interlock and travel as one; what is their combined speed before brakes are applied?

Solution: The individual momentums and masses of both vehicles can be combined, and the resulting velocity determined as follows:

$$\frac{m_1V_1 + m_2V_2}{m_T} = V_T = \frac{(11,000\,\text{lbs})(60\,\text{mph}) + (7,000\,\text{lbs})(40\,\text{mph})}{18,000\,\text{lbs}} = 52.2\,\text{mph}$$

Important: Notice the units in the equation do not match those listed above? That is because there is no need to convert here as long as the same units are used (i.e., lbs and lbs, and mph and mph). Try doing all the conversions and see for yourself.

5.4 Work

In mechanics, work is the amount of energy transferred by a force acting through a distance,

$$W = Fs \tag{110}$$

where

W = work, ft-lbs

F = force, lbs

s = distance, ft

Problem: A 10 lb weight is lifted 50 feet. How much work is required?

Solution: We simply plug the values into equation (110) and find:

$$W = Fs = (10\,\text{lbs})(50\,\text{ft}) = 500\,\text{ft-lbs}$$

Note: Ft-lbs can be easily converted to other units such as joules, watts-sec, calories, Btus, etc.

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5.5 Moment of Force

Moment of force can be thought of as a rotational force resulting from a force acting some distance from a point. When balanced by an opposing but equal moment, this can be written as:

$$F_1D_1 = F_2D_2 \tag{111}$$

where

F_n = force n, lbs

D_n = distance n, feet

Problem: The moment force is the principle behind levers. Assume a 55 gallon drum contains about 400 lbs of fluid and you want to lift the drum to place a pad under it. You connect a sling to the drum and attach it to a 10 foot long steel bar. The steel bar is then placed over a pivot point such that there is 2 feet of bar between the pivot point and the sling, and the remaining 8 feet is on the other side of the pivot point. What force must be applied to the end of the steel bar to lift the drum?

Solution: We can use equation (111) to calculate the force required and see the mechanical advantage of levers.

$$F_1D_1 = F_2D_2 = (400\,\text{lbs})(2\,\text{ft}) = (F_2)(8\,\text{ft})$$

Solving for F_2 yields:

$$F_2 = \frac{(400\,\text{lbs})(2\,\text{ft})}{(8\,\text{ft})} = 100\,\text{lbs}$$

In this case it takes a force ¼ the weight to lift the weight.

5.6 Friction

Friction is the force resisting the relative motion of two objects sliding against each other. Mathematically, this relationship can be written as:

$$F = \mu N \tag{112}$$

where

F = frictional force, lb-force

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μ = coefficient of friction, nondimensional

N = the normal (perpendicular) force, lb-force

Problem: A pallet with a load weighs 350 lbs. If the coefficient of friction is 0.65, what horizontal force must be applied to slide the pallet?

Solution: The horizontal force must be equal to or greater than the frictional force. Applying equation (112) and substituting values:

$$F = \mu N = (0.65)(350\,\text{lbs}) = 227.5\,\text{lbs}$$

5.7 Potential Energy

In mechanics, potential energy is the energy stored in an object due to its position. This can be written as:

$$P.E. = mgh \tag{113}$$

where

$P.E.$ = potential energy, ft-lbs

m = mass, slugs

g = gravitational acceleration, ft/sec^2

h = height, ft

Note from equation (108) above, $W = mg$, so we can write:

$$P.E. = Wh \tag{114}$$

Problem: An air conditioning unit is being lifted to the roof of a new building. The unit weighs 750 lbs and the roof is located 45 feet above grade. What is the potential energy of the unit as it reaches roof level?

Solution: Applying equation (114) results in:

$$P.E. = Wh = (750\,\text{lbs})(45\,\text{ft}) = 33{,}750\,\text{ft-lbs}$$

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5.8 Hooke's Law and the Potential Energy of a Spring

In mechanics, Hooke's law states the extension of a spring is in direct proportion to the force acting on it as long as this load does not exceed the elastic limit. This can be written:

$$F = -kx \tag{115}$$

where

F = force on spring, lbs

k = spring constant, lbs/ft

x = distance spring is changed, ft

The potential energy stored in a spring can be derived in the following manner.

Recalling equation (110) above, the work done by the spring force (F) over some displacement (s) is given by $W = Fs$. The work stored in the spring is its potential energy.

Thus, we can write:

$$P.E. = -\int F dx = -\int -k\left(x - x_o\right)dx = \tfrac{1}{2}k\left(x - x_o\right)^2 + C \tag{116}$$

Setting $C = 0$ so that $P.E.$ is zero at $x = x_o$ and making the equilibrium position zero ($x_o = 0$) simplifies Equation (116) to:

$$P.E. = \frac{kx^2}{2} \tag{117}$$

where

$P.E.$ = potential energy in spring, ft-lbs

k = spring constant, lbs/ft

x = distance spring is changed, ft

Problem: A spring with a constant of 15,000 lbs/ft is compressed 3 inches. What is the potential energy stored in the compressed spring?

Solution: Converting 3 inches to 0.25 ft and substituting the values into equation (117)

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yields:

$$P.E. = \frac{kx^2}{2} = \frac{(15{,}000\,\text{lbs/ft})(0.25\,\text{ft})^2}{2} = 469\,\text{ft-lbs}$$

5.9 Kinetic Energy

The kinetic energy of an object is the energy which it possesses due to its motion. Mathematically this can be expressed as:

$$K.E. = \frac{mv^2}{2} \tag{118}$$

where

K.E. = kinetic energy, ft-lbs

m = mass of moving object, slugs

v = velocity of object, ft/sec

Problem: A forklift weighs 3980 lbs; what is its kinetic energy when traveling at 10 mph?

Solution: First, convert weight in pounds to slugs and speed in mph to ft/sec, and then use equation (118) to calculate kinetic energy.

$$K.E. = \frac{mv^2}{2} = \frac{\left(\frac{3980\,\text{lbs}}{32.2\,\text{ft/sec}^2}\right)\left[\left(10\frac{\text{miles}}{\text{hr}}\right)\left(5280\frac{\text{ft}}{\text{mile}}\right)\left(\frac{1\text{hr}}{3600\,\text{sec}}\right)\right]^2}{2} = 13{,}294\text{ft-lbs}$$

5.10 Rectilinear Motion

In simple terms, rectilinear motion refers to the motion of objects along straight line without consideration of outside forces. Within rectilinear motion; distance, velocity, and acceleration are related by the following equations:

$$v = v_o + at \tag{119}$$

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$$s = v_o t + \frac{at^2}{2} \qquad (120)$$

$$v^2 = v_o{}^2 + 2as \qquad (121)$$

where

s = distance, ft

v = velocity, ft/sec

a = ft/sec^2

t = time, sec

Problem: A worker is attempting to throw a small bundle of rope to a 60 ft high platform. Assume the worker can throw the rope straight up at 20 mph from a starting height of 6 feet. Will the rope make it to the platform?

Solution: We can solve this two ways. First, use equation (119) to calculate the time (*t*) the rope travels up. Note that when the rope reaches its highest point, its velocity will be zero. So:

- v = 0
- v_o = 20 mph = 58.67 ft/sec

$$0 = 58.67\text{ft/sec} + (-32.2\text{ft/sec}^2)\cdot t$$

Solving for *t* leads to *t* = 1.82 seconds.

Then using equation (120), we can solve for the distance traveled,

$$s = (58.67\text{ft/sec})(1.82\sec) + \frac{(-32.2\text{ft/sec}^2)(1.82\sec)^2}{2}$$

Solving for *s* leads to *s* = 53.4 feet.

We must add 6 feet, so the height reached by the rope is 53.4 ft + 6 ft = 59.4 feet. So he just barely misses it. Of course this ignores air resistance and assumes a perfect vertical path.

Note that you can also solve this more directly by using equation (121) and setting the final velocity to zero:

$$0 = \left(58.67\,\text{ft/sec}\right)^2 + 2\left(32.3\,\text{ft/sec}^2\right)\cdot s$$

Solving for *s* leads to *s* = 53.4 feet.

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Problem: When the worker is finished working on the platform, he drops the rope from the platform. How fast is the bundle of rope moving when it hits the floor? Assume he drops the rope from about 3 feet above the platform floor.

Solution: The total distance the rope will fall is 60 ft + 3 ft = 63 ft. This time the initial velocity is zero. Again using equation (121) and solving for v:

$$v^2 = \left(0 \text{ ft/sec}\right)^2 + 2\left(32.3 \text{ft/sec}^2\right) \cdot 63 \text{ft}$$

Solving for v leads to v = 63.7 ft/sec = 39.4 mph

Getting struck by a bundle of rope traveling at nearly 40 mph can cause serious injury. Note that in the above calculations, the weight or size of the rope was not required.

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Hydrostatics and Hydraulics

6 Hydrostatics and Hydraulics

6.1 Pressure and Force

Hydrostatics and Hydraulics refers to properties of water at rest and in motion. One basic relationship is that which relates pressure and force; this is given by the following equation:

$$P = \frac{F}{A} \tag{122}$$

where

P = pressure, lbs/ft^2

F = force, lbs

A = area, ft^2

Problem: A tanks holds 3000 pounds of quench water. If the tank has a square bottom and each side is 4 feet long, what is the pressure exerted on the base of the tank?

Solution:

$$P = \frac{F}{A} = \frac{3000\,\text{lbs}}{(4\,\text{ft})(4\,\text{ft})} = \left(187.5\frac{\text{lbs}}{\text{ft}^2}\right)\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right) = 1.3\,\text{psi}$$

6.1.1 Static Pressure

One cubic foot of water weighs 62.4 lbs (i.e., 62.4 lbs/ft^3). Therefore a column of water measuring 1 foot high creates a pressure of:

$$\frac{62.4\,\text{lbs}}{144\,\text{in}^2} = 0.433\,\text{psi} \tag{123}$$

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To determine the pressure (in psi) exerted by a column of water of any height, simply multiply equation (123) by the height, in feet, or:

$$P_{psi} = 0.433\,h \tag{124}$$

or, for units of pound- per-square-foot (psf):

$$P_{psf} = 62.4\,h \tag{125}$$

If we call the specific weight of water (62.4 lb/ft^3), w, we can write:

$$P_{psf} = w\,h \tag{126}$$

Solving for *h* leads to:

$$h = \frac{P}{w} \tag{127}$$

The *h* in equation (127) is known as the pressure head, and has units of feet. This is the net or normal pressure; that is pressure exerted against the side of a container (e.g., pipe) without flow. Since it represents a pressure head, it is usually written as:

$$h_P = \frac{P}{w} \tag{128}$$

Problem: What pressure would be measured at the base of a fire standpipe in a 5 story high-rise? Assume each floor is 12 feet high.

Solution: We can re-arrange equation (128) and substitute to find:

$$P = h_P w = (5\,\text{stories})(12\,\text{ft/story})\left(62.4\,\text{lbs/ft}^3\right) = 3744\,\text{lbs/ft}^2$$

Or simply use equation (124) to find the answer directly in psi:

$$P_{psi} = 0.433\,h = 0.433(5\,\text{stories})(12\,\text{ft/story}) = 26\,\text{psi}$$

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6.1.2 Velocity Pressure

Velocity pressure, as the name implies, is the pressure due to moving water. The velocity produced in a mass of water by the pressure acting on it is the same as if the same mass of water were to fall freely from some height, *h*, that creates an equivalent pressure. This can be shown as follows:

Recall the equation for kinetic energy:

$$K.E. = \frac{mv^2}{2} \tag{129}$$

And the equation for potential energy:

$$P.E. = mgh \tag{130}$$

When the potential energy of water at some height is turned into kinetic energy as it falls, equations (129) and (130) can be set equal:

$$mgh = \frac{mv^2}{2} \tag{131}$$

Solving for h (and since it is the velocity head), labeling it as h_V, leads to:

$$h_v = \frac{V^2}{2g} \tag{132}$$

This can be solved for the velocity to find:

$$V = \sqrt{2gh_v} \tag{133}$$

This equation is known as Torricelli's law, or Torricelli's theorem (not to be confused with Torricelli's equation).

> Problem: A 2-1/2 inch valve is opened at the base of a large water storage tank. If the surface of the water in the tank is 50 feet above the open valve. What is the velocity of the water exiting the open valve?
>
> Solution: We can use equation (133) and substitute the value for the height and the gravitational acceleration (32.2 ft/sec^2):

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$$V = \sqrt{2gh_v} = \sqrt{(2)\left(32.2\,\text{ft/sec}^2\right)(50\,\text{ft})} = 56.7\,\text{ft/sec}$$

Note that as the water level drops, so does the velocity. Also note that the size of the opening does not affect velocity. However, since we know the velocity and the size of the opening, we can also solve for the actual flow (e.g., gpm).

6.2 Bernoulli's Theorem

Equations (128) and (132) are part of Bernoulli's theorem. Bernoulli's theorem is an expression that relates, through conservation of energy, the pressure, velocity and elevation (height) of the steady flow of an incompressible, non-viscous fluid. Remember, "fluids" includes liquids and gases; so Bernoulli's theorem also applies to gases that can be considered incompressible (i.e., the density can be considered constant). This theorem is also known as Bernoulli's equation or Bernoulli's law, and is shown here:

$$\frac{V_1^2}{2g} + \frac{P_1}{w} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{w} + Z_2 + h_{1-2} \tag{134}$$

where

V = Velocity, ft/sec

g = gravitational acceleration, ft/sec^2

P = Pressure, lbs/ft^2

w = Specific weight, lbs/ft^3

Z = Elevation, ft

$h_{1\text{-}2}$ = energy (head) lost between locations 1 and 2, ft

Notice that each group of variables (e.g., $V^2/2g$) has units of feet and is referred to as "head."

Problem: A fire truck draws water from a pond that is 6 feet below the fire truck. It then pumps the water up to a fire that is 15 feet higher through 250 feet of 2 inch hose to a 1-1/2 inch nozzle that discharges 100 gpm into the fire. Assume the friction losses in the hoses total 30 psi. What pressure does the pump need to add to move the water from the pond to the fire? (Note: 1 gallon of water = 0.1337 ft^3)

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Solution: First, we need to change the friction losses from psi to head, using equation (124):

$$P_{psi} = 0.433\,h$$

$$\frac{P_{psi}}{0.433} = h = \frac{30\,\text{psi}}{0.433} = 69.3\,\text{ft}$$

Next, the pond is at zero velocity, but the water discharging from the nozzle has a velocity. We find this by first finding the velocity of the water exiting the nozzle, then using equation(132). For this we need the area of the nozzle:

$$A = \pi\frac{d^2}{4} = \pi\frac{2^2}{4} = 3.14\,\text{in}^2 = \left(3.14\,\text{in}^2\right)\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right) = 0.0218\,\text{ft}^2$$

Now we can find the velocity from the continuity equation (Q=AV):

$$V = \frac{Q}{A} = \left(\frac{100\,\text{gpm}}{0.0218\,\text{ft}^2}\right)\left(\frac{0.1337\,\text{ft}^3}{1\,\text{gallon}}\right) = 613.3\,\text{ft/min} = 10.22\,\text{ft/sec}$$

From equation (132):

$$h_v = \frac{V^2}{2g} = \frac{\left(10.22\,\text{ft/sec}\right)^2}{2\left(32.2\,\text{ft/sec}^2\right)} = 1.62\,\text{ft}$$

We can now apply Bernoulli's equation(134):

$$\frac{V_1^2}{2g} + \frac{P_1}{w} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{w} + Z_2 + h_{1-2}$$

$$0 + 0 + (-6\,\text{ft}) = 1.62\,\text{ft} + \frac{P_2}{w} + 15\,\text{ft} + 69.3\,\text{ft}$$

or

$$\frac{P_2}{w} = -\left(91.92\,\text{ft}\right)$$

Since this is what the overall pressured drop (in feet) it is also what the pump must add to compensate. To convert to psi:

$$\left(91.92\,\text{ft}\right)\left(62.4\,\text{lb/ft}^3\right)\left(\frac{1\text{ft}^2}{144\text{in}^2}\right) = 39.6\,\text{psi}$$

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6.3 Water Flow in a Pipe

The velocity pressure in a pipe with a given flow (e.g., gallons per minute, gpm) can be derived as follows:

Recalling the continuity equation:

$$Q = A \cdot V \tag{135}$$

or

$$V = \frac{Q}{A} \tag{136}$$

where

Q = volumetric flow rate, ft^3/sec

A = cross-sectional area, ft^2

V = velocity, ft/sec

Converting gallons per minute to cubic feet per seconds,

$$\left(\frac{\text{gallons}}{\text{minute}}\right)\left(\frac{1}{60\,\text{sec/min}}\right)\left(\frac{1}{7.48\,\text{gal/ft}^3}\right) \tag{137}$$

and converting the cross sectional area of a pipe in square inches to square feet

$$\left(\frac{\pi d^2}{4}\right)\left(\frac{1}{144\,\text{in}^2/\text{ft}^2}\right) \tag{138}$$

where

d = diameter of pipe, inches

Combining equations (136), (137), and (138) leads to:

$$V = \frac{Q}{A} = \left(\frac{(\text{gpm})(4)(144)}{(60)(7.48\,\text{gal/ft}^3)(\pi d^2)}\right) = \frac{(0.4085)(\text{gpm})}{(d^2)} \tag{139}$$

Recall equation (132), and substituting the value of V just derived leads to:

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$$h_v = \frac{V^2}{2g} = \frac{\left(\frac{0.4085 \cdot gpm}{d^2}\right)^2}{2g} = \frac{gpm^2}{386d^4} \tag{140}$$

Recalling equation (124), and substituting h_v just derived, leads to:

$$P(psi) = 0.433\frac{gpm^2}{386d^4} = \frac{gpm^2}{891d^4} \tag{141}$$

which is typically written with Q substituted for gpm

$$P_V = \frac{Q^2}{891d^4} \tag{142}$$

Problem: What is the velocity pressure created by water flowing at 100 gpm in a nominal 2 inch pipe? Assume the actual internal diameter pipe is 2.07 inches.

Solution: Substitute the flow and pipe diameter values into equation (142):

$$P_V = \frac{Q^2}{891d^4} = \frac{(100\,\text{gpm})^2}{891 \cdot (2.07\,\text{in})^4} = 0.61\,\text{psi}$$

Remember the 891 is a conversion, so units must be in gpm and inches, and resulting velocity pressure is in psi.

6.3.1 Flow – Pressure Relationships

For flow in a pipe with fixed diameter, equation (142) can be written:

$$P = \frac{Q^2}{C} \tag{143}$$

where C is a constant (due to the diameter being fixed). This equation can also be written:

$$\frac{Q_1^2}{P_1} = C = \frac{Q_2^2}{P_2} \tag{144}$$

which can be rearranged to yield:

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$$\frac{Q_1}{Q_2} = \frac{\sqrt{P_1}}{\sqrt{P_2}} = \sqrt{\frac{P_1}{P_2}} \tag{145}$$

Problem: A water supply system to a series of emergency showers is designed and the water flow and pressure are known. A decision is then made to increase the required flow by a 25% safety factor. What increase in pressure is required?

Solution: Since we are concerned with ratios, the exact flow and pressures are not required. We do know $Q_2 = Q_1 \cdot (1.25)$. We can re-arrange equation (145) and substitute as follows:

$$P_2 = \left(\frac{Q_2}{Q_1}\right)^2 P_1 = \left(\frac{1.25}{1}\right)^2 P_1$$

$$P_2 = 1.56 \cdot P_1$$

So we can see increasing the flow by 25% requires the pressure be increased by 56%.

Another useful equation is one that relates the flow from an orifice (e.g., a fire sprinkler) due to the pressure at the orifice. Equation (143) can be written as:

$$P = \frac{Q^2}{C} = \left(\frac{Q}{K}\right)^2 \tag{146}$$

where

K = constant based on the orifice

Equation (146) is commonly applied in the form:

$$Q = K\sqrt{P} \tag{147}$$

where

Q = water flow, gpm

K = orifice factor, gpm/psi$^{1/2}$

P = pressure, psi

Problem: The pressure in a sprinkler supply pipe is 25 psi at the location of a sprinkler.

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What is the expected flow in gpm for that sprinkler? Assume K=5.6 gpm/psi$^{1/2}$ (a common K-factor for fire sprinklers).

Solution: Applying equation (147): $Q = K\sqrt{P} = 5.6\sqrt{25} = 28\,\mathrm{GPM}$

6.3.2 Hazen-Williams Formula

The design or evaluation of hydraulic systems typically requires the calculation of pressure losses due to friction as water flows through a section of pipe. This is typically accomplished using the Hazen-Williams formula:

$$P_d = \frac{4.52Q^{1.85}}{C^{1.85}d^{4.87}} \qquad (148)$$

where

P_d = pressure drop, psi/ft

4.52 = constant based on pressure losses per-foot

Q = flow, gpm

C = Hazen-Williams coefficient, this is related to the roughness of the piping

d = pipe diameter, in

Notice the atypical power values (i.e., 1.85 and 4.87) used in equation (148). This is due to the Hazen-Williams formula being an empirical formula. An empirical formula is a mathematical equation that predicts observed results, but is derived from experiment and not directly from first principles.

Problem: A new 8 inch (nominal) cast iron water supply line, 500 feet in length, is run to a new building. What is the friction loss when 1000 gpm is flowing through the pipe? Assume a Hazen-Williams coefficient of 120 and an interior diameter of 8.3 inches.

Solution: First, we use equation (148) to calculate the friction loss per foot, and then multiply that by the total length.

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$$P_d = \frac{4.52Q^{1.85}}{C^{1.85}d^{4.87}} = \frac{4.52\left(1000\,\text{gal}\right)^{1.85}}{\left(120\right)^{1.85}\left(8.3\,\text{in}\right)^{4.87}} = 0.0076\,\text{psi/ft}$$

$$P_{total} = \left(500\,\text{ft}\right)\left(0.0076\,\text{psi/ft}\right) = 3.82\,\text{psi}$$

Another hydraulic formula that is commonly used to evaluate water supplies is the following expression which relates changes in water flow due to changes in residual pressures. Static pressure is the pressure measured on a water supply when there is no water flowing and the residual pressure is the pressure remaining when there is water flow.

$$Q_2 = Q_1\left[\frac{\left(S-R_2\right)^{0.54}}{\left(S-R_1\right)^{0.54}}\right] \tag{149}$$

where

Q_1 = flow at residual pressure R_1, gpm

Q_2 = flow at residual pressure R_2, gpm

S = static pressure on the water supply system, psi

R_1 = residual pressure when flowing Q_1, psi

R_2 = residual pressure when flowing Q_2, psi

Problem: A pressure gauge is placed on a fire hydrant and the pressure recorded with no water flowing is 80 psi. The next closest hydrant is opened and a Pitot tube is used to measure and calculate a flow of 3000 gpm; the pressure gauge at the first hydrant now reads 58 psi. A second hydrant is partially opened and the pressure gauge on the first hydrant now shows 50 psi. Without having to use a Pitot tube at both flowing hydrants, calculate the total flow from both hydrants.

Solution: We know the static pressure on the water supply system at this location is 80 psi. We also know that when flowing 3000 gpm, the residual pressure is 58 psi. We also know when a second hydrant is opened; the residual pressure drops to 50 psi. From this, we can use equation (149) to find the new (combined) water flow:

$$Q_2 = Q_1\left[\frac{\left(S-R_2\right)^{0.54}}{\left(S-R_1\right)^{0.54}}\right] = 3000\,\text{gpm}\left[\frac{\left(80\,\text{psi-}50\,\text{psi}\right)^{0.54}}{\left(80\,\text{psi-}58\,\text{psi}\right)^{0.54}}\right] = 3547\,\text{gpm}$$

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Heat Transfer

7 Heat Transfer

Heat transfer is the transfer of energy between material bodies as a result of temperature differences. There are three modes of heat transfer: conduction, convection and radiation. The following equations present simple forms of the three heat transfer modes.

7.1 Conduction

$$\frac{q}{A} = k\frac{(T_1 - T_2)}{(x_1 - x_2)} \tag{150}$$

where

q = heat transferred, Btu/hr

A = area through which heat is conducted, ft^2

k = thermal conductivity, Btu/hr-ft-^oF

T_1 = temperature at location x_1, ^oF

T_2 = temperature at location x_2, ^oF

x_1 = location of T_1, ft

x_2 = location of T_2, ft

7.2 Convection

$$\frac{q}{A} = h(T_w - T_\infty) \tag{151}$$

where

q = heat transferred, Btu/hr

A = area through which heat is conducted, ft^2

h = convective heat transfer coefficient, Btu/hr-ft^2-^oF

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T_w = temperature solid surface, ^oF

T_∞ = temperature of fluid (e.g., air) in which energy is transferred, ^oF

Note: The true definition of fluids includes liquids and gases.

7.3 Radiation

$$\frac{q}{A} = \sigma\left(T_1^4 - T_2^4\right) \quad (152)$$

where

q = heat transferred, Btu/hr

A = area through which heat is conducted, ft^2

σ = Stephan-Boltzman constant, 0.1714 10^{-8} Btu/hr-ft^2 -$^oR^4$

T_w = temperature of solid surface, ^oR

T_∞ = temperature of fluid (e.g., air) which energy is transferred, ^oR

Important: For radiation heat transfer calculations, temperatures are absolute and raised to the fourth power.

Problem: There are two rooms separated by a 6 inch concrete wall. In one room there is a fully developed fire and the average room gas temperature is 1000^{o}F. The other room is large and the room temperature is maintained at 70^{o}F. Calculate the wall surface temperatures of the separating wall. Assume h = 1.5 Btu/hr-ft^2-^{o}F in the fire room, h = 0.7 Btu/hr-ft^2-^{o}F in the other room, and the thermal conductivity of the concrete is 0.45 Btu/hr-ft-^{o}F. Assume radiation gains and losses can be ignored. The energy balance across the wall can be written as:

$$h_f\left(T_f - T_{Iw}\right) = k\frac{\left(T_{Iw} - T_{Ow}\right)}{\left(x_1 - x_2\right)} = h_\infty\left(T_{Ow} - T_\infty\right)$$

where

q = heat transferred, Btu/hr
A = area through which heat is conducted, ft^2
h_f = convective heat transfer coefficient, Btu/hr-ft^2-^{o}F
T_f = temperature in fire room, ^{o}F
T_{Iw} = temperature of wall surface in fire room, ^{o}F
k = thermal conductivity of concrete wall, Btu/hr-ft-^{o}F
T_{Ow} = temperature of wall surface in other room, ^{o}F
x_1-x_2 = wall thickness, ft
h_∞ = convective heat transfer coefficient, Btu/hr-ft^2-^{o}F
T_∞ = temperature in other room, ^{o}F

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Entering what we know leads to:

$$1.5\left(1000 - T_{Iw}\right) = 0.45\frac{\left(T_{Iw} - T_{Ow}\right)}{\left(0.5\right)} = 0.7\left(T_{Ow} - 70\right)$$

There are a few methods to solve this. The direct approach is to isolate one variable and then substitute. Another approach is to use the Excel Add-in called Solver. Either method used should result in:

T_{Iw} = 806.6 ^{o}F
T_{Ow} = 484.4 ^{o}F

The calculated wall surface temperature of 484.4 ^{o}F is very high and suggests radiation losses should be considered. The heat losses from the wall surface from convective and radiative heat losses can be found as follows:

$$\frac{q}{A} = 0.7\left(484.4 - 70\right) = 290.1 \text{ Btu/hr-ft}^2$$

and

$$\frac{q}{A} = 0.1714 \text{ x}10^{-8}\left((484.4 + 459.69)^4 - (70 + 459.69)^4\right) = 1226.7\text{Btu/hr-ft}^2$$

This demonstrates the radiative losses from the surface of the other wall is over four times greater than the convective losses, so the original assumption was incorrect (remember always to confirm assumptions are appropriate). To add the radiative losses, the above equation can be written to include the radiative losses from the surface of the second room.

$$1.5\left(1000 - T_{Iw}\right) = 0.45\frac{\left(T_{Iw} - T_{Ow}\right)}{\left(0.5\right)} = 0.7\left(T_{Ow} - 70\right) + 0.1714 \text{ x}10^{-8}\left((T_{Ow} + 459.69)^4 - (70 + 459.69)^4\right)$$

The Excel Add-in called Solver was used to solve this set of equations and resulted in:

T_{Iw} = 718.3 ^{o}F
T_{Ow} = 248.9 ^{o}F

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Ventilation

8 Ventilation

8.1 Conservation of Mass (the Continuity Equation)

A basic concept when evaluating the flow of a gas in a system (e.g., air in a duct) is conservation of mass. This states that the mass flow rate of a gas at one point in a stream is equal to the mass flow rate at any other location (assuming no additions or losses). This is also known as the "continuity equation" and can be written as:

$$A_1 \cdot V_1 \cdot \rho_1 = A_2 \cdot V_2 \cdot \rho_2 \qquad (153)$$

where

A_n = cross-sectional area at location n, ft^2

V_n = velocity at location n, ft/min

ρ_n = density of gas at location n, lbs/ft^3

In most applications of building ventilation, it can be assumed that ρ behaves as a constant (i.e., $\rho_1 = \rho_2$) and equation (153) can be written as:

$$Q_1 = A_1 \cdot V_1 \qquad (154)$$

where

Q_1 = volumetric flow, ft^3/min

Note that equation (154) can be rearranged to show:

$$A_1 = Q_1 / V_1 \qquad (155)$$

and

$$V_1 = Q_1 / A_1 \qquad (156)$$

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For constant volumetric flow, we can write:

$$Q_1 = Q_2 \tag{157}$$

or

$$A_1 V_1 = A_2 V_2 \tag{158}$$

Through simple rearrangement of Equation (158), any one of the four variables can be found if the other three are known. For example, solving for V_2 results in Equation (158) being written as:

$$V_2 = \frac{A_1 V_1}{A_2} \tag{159}$$

Problem: The design of a section of duct has air velocities that are too high. To reduce the velocity in half, what change to the duct cross-sectional area would be required?

Solution: We can see from equation (158) that to decrease the velocity by half, the duct area must be doubled.

8.2 Conservation of Energy

The energy in a ducted ventilation flow stream (assuming no losses) can be written as:

$$TP = VP + SP \tag{160}$$

where

TP = total pressure, inches of water column (also written as in. wc)

VP = velocity pressure, in. wc

SP = static pressure, in. wc

TP represents to total energy, or "head" in a flow stream at any location. *VP* represents the pressure due to movement (it is always positive) and *SP* represents the pressure of the fluid or gas exerted in all directions.

Due to conservation of energy (i.e., *TP* remains constant), Equation (160) can be re-written as:

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$$SP_1 + VP_1 = SP_2 + VP_2 + h_L \tag{161}$$

where

TP_n = total pressure at location n, inches of water column (also written as in. wc)

VP_n = velocity pressure at location n, in. wc

SP_n = static pressure at location n, in. wc

h_L = head (energy) loss from location 1 to location 2, in. wc

Problem: Measurements made at two ends of a section of ductwork showed a total pressure at one location of 2.5 in. wc, and 2.25 in wc. at the other end. What is the head loss across the section of ductwork?

Solution: Combining equations (160) and (161) provides:

$$TP_1 = TP_2 + h_L$$

$$h_L = TP_1 - TP_2 = (2.5 \text{ in.wc}) - (2.25 \text{ in.wc}) = 0.25 \text{ in.wc}$$

Velocity pressure is always positive, and an average duct velocity pressure can be found using the following expression:

$$VP_{ave} = \left(\frac{\sqrt{VP_1} + \sqrt{VP_2} + \ldots + \sqrt{VP_n}}{n} \right)^2 \tag{162}$$

where

VP_{ave} = average velocity pressure, in. wc

VP_n = velocity pressure n, in. wc

n = number of velocity pressure readings

We will now explore some common applications of equation (161) and the law of conservation of energy and mass.

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Problem: Three velocity pressures are sampled across a duct and the following data recorded: 0.75, 1.0 and 0.95 in. wc. What is the average velocity pressure at that location?

Solution:

$$VP_{ave} = \left(\frac{\sqrt{VP_1} + \sqrt{VP_2} + \ldots + \sqrt{VP_n}}{n} \right)^2 = \left(\frac{\sqrt{0.75} + \sqrt{1.0} + \sqrt{0.95}}{3} \right)^2 = 0.90 \text{ in.wc}$$

8.3 Derivation of the Fundamental Duct Flow Equations

A very common equation related to air flow in a duct is typically written as:

$$V = 4005\sqrt{VP} \qquad (163)$$

where

V = velocity of air, ft/min

4005 = a constant based on air flowing at standard temperature and pressure (STP)

VP = velocity pressure, in. wc

This equation can be derived as follows (first we will derive it for any gas and then for air).

Constants Use caution whenever you see a constant in an equation (e.g., 4005 in equation (163)). This frequently means the equation uses a set type of units. Using the wrong units will lead to incorrect calculations.

From Torricelli's law, see equation (133), the velocity of a gas created by the velocity pressure (head) of a column of gas can be written as:

$$V = \sqrt{2gh_{gas}} \qquad (164)$$

where

V = velocity of gas, ft/sec

g = gravitational acceleration, ft/sec^2

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h_{gas} = elevation head, ft of gas

Since the elevation head in equation (164) is in feet of gas and we want to use water column, we need to convert equation (164) from gas to water. We can do this with this relationship:

$$\rho_{gas} h_{gas} = \rho_{water} h_{water} \tag{165}$$

where

ρ_{gas} = density of gas (at STP), lbs/ft^3

h_{gas} = elevation head, ft of gas

ρ_{water} = density of water (at STP), lbs/ft^3

h_{water} = elevation head, ft of water

Equation (165) can be re-written as:

$$h_{gas} = \frac{\rho_{water} h_{water}}{\rho_{gas}} \tag{166}$$

Note that we also need to change from feet of gas to inches of water head; this is done by the following conversion (which allows the substitution of VP for h_{water}):

$$h_{gas} = \frac{1 \cdot VP \cdot \rho_{water}}{12 \cdot \rho_{gas}} \tag{167}$$

Combining equation (164) and (167), and converting from seconds to minutes (that's the 60 in front of the radical), leads to:

$$V = 60 \cdot \sqrt{\frac{2g \cdot \rho_{water} VP}{12 \cdot \rho_{gas}}} \tag{168}$$

Substituting values for g, ρ_{water}, and maintaining ρ_{gas} for now, results in:

$$V = 1096 \sqrt{\frac{VP}{\rho_{gas}}} \tag{169}$$

where

V = velocity, ft/min

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VP = velocity pressure (head), in. wc

ρ_{gas} = density of gas (at STP), lbs/ft^3

Problem: What is the velocity in a duct when the velocity pressure recorded is 0.90 in.wc? The duct carries nitrogen at normal temperature and pressure; assume a density of 0.073 lbs/ft^3.

Solution:

$$V = 1096\sqrt{\frac{VP}{\rho_{gas}}} = 1096\sqrt{\frac{0.90}{0.073}} = 3{,}848 \text{ cfm}$$

Equation (169) can be used to find the velocity of any gas at STP flowing in a duct. However, we are frequently concerned with air movement in ventilation systems. When the value for the density of air at STP (0.075 lb/ft^3) is substituted for ρ_{gas}, we find:

$$V = 4005\sqrt{VP} \tag{170}$$

where

V = velocity or air, ft/min

VP = velocity pressure (head), in. wc

Problem: What is the velocity in an air duct when the velocity pressure recorded is 0.90 in.wc? Assume standard air conditions.

Solution: Since we are dealing with air, we can use equation (170):

$$V = 4005\sqrt{VP} = 4005\sqrt{0.9} = 3799 \text{ cfm}$$

Note that this is very close to the value for nitrogen just calculated above. Since air is 79% nitrogen, its density is very close to air.

Note that equation (170) only applies to air at standard temperature and pressure (STP). If the air is not at standard temperature and pressure, this must be accounted for in equation (170). Once again, a conversion is required.

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8.3.1 Density Correction Factor

Density of gases is a function of temperature and pressure. When conditions vary from standard temperature and pressure (STP), a *density factor* must be applied to account for this variation. This is shown here:

$$\rho_{Actual} = \rho_{STP} \cdot df \tag{171}$$

where

ρ_{Actual} = density of gas (at some temperature and pressure), lbs/ft^3

ρ_{STP} = density of gas (at STP), lbs/ft^3

df = density factor, non-dimensional

Dimensionless Number A dimensionless number is a quantity without a physical unit; a pure number. Such a number is typically defined as a product or ratio of quantities that might have units individually, but which cancel out when taken in combination. They are very useful in calculations as they are not scale or unit dependant.

For industrial hygiene and safety applications, the density factor is typically calculated as follows:

$$df = \left(\frac{530}{T+460}\right)\cdot\left(\frac{BP}{29.92}\right) \tag{172}$$

where

T = temperature of gas, ^{o}F

BP = barometric pressure, inches of Mercury (in. Hg)

Problem: A location is 1000 feet above sea level, and the local barometric pressure is 28.86 mmHg, and the temperature is 90 ^{o}F. What is the density correction factor for these conditions?

Solution:

$$df = \left(\frac{530}{T+460}\right)\cdot\left(\frac{BP}{29.92}\right) = \left(\frac{530}{90+460}\right)\cdot\left(\frac{28.86}{29.92}\right) = 0.93$$

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Equation (172) can be derived from the ideal gas law (see section 4.1 above):

$$P \cdot Vol = n \cdot R \cdot T \tag{173}$$

where

P = absolute pressure of the gas, atm

Vol = volume of gas, liters (l)

n = amount of gas, moles

R = gas constant, 0.082 l-atm/moles-K

These are the typical units used in safety and industrial hygiene applications. However, other applications (or simply personal preference) may employ different units.

For a gas at two varying conditions, Equation (173) can be written as:

$$\frac{P_1 Vol_1}{nRT_1} = \frac{P_2 Vol_2}{nRT_2} \tag{174}$$

For n and R being constant, equation (174) can be written:

$$\frac{P_1 Vol_1}{T_1} = \frac{P_2 Vol_2}{T_2} \tag{175}$$

We know that density is a measure of the amount of a gas in a given volume. For a fixed amount (mass) of gas, the volume is inversely-proportional to the density. This can be written as:

$$Vol \propto \frac{1}{\rho} \tag{176}$$

Proportionality Symbol $\propto$ In mathematics, two quantities are said to be proportional if each of the quantities is a constant multiple of the other. There is no specific relationship given; in fact the lack of specific detail is the reason the Proportionality Sign is used.

Combining Equations (175) and (176) results in:

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$$\frac{P_{Actual}}{\rho_{Actual} T_{Actual}} = \frac{P_{STP}}{\rho_{STP} T_{STP}} \tag{177}$$

which can be rearranged to

$$\frac{\rho_{Actual}}{\rho_{STP}} = \frac{T_{STP} P_{Actual}}{T_{Actual} P_{STP}} = df \tag{178}$$

Recall that Equations (169) and (170) are based on standard temperature and pressure (STP) conditions. We can use the density factor (178) to modify those equations (and others) to account for conditions other than standard temperature and pressure, as seen here:

$$V = 1096\sqrt{\frac{VP}{df \cdot \rho_{gas}}} \tag{179}$$

$$V = 4005\sqrt{\frac{VP}{df}} \tag{180}$$

Remember that at standard temperature and pressure (STP), $df = 1.0$.

Problem: In the previous sample problem, the density factor calculated was 0.93. Compare the velocity of air with that density factor and conditions at STP (i.e., df = 1.0). Assume a velocity pressure of 1.0 in.wc.

Solution:

$$V = 4005\sqrt{\frac{VP}{df}} = 4005\sqrt{\frac{1.0}{0.93}} = 4153 \text{ ft/min}$$

$$V = 4005\sqrt{\frac{VP}{df}} = 4005\sqrt{\frac{1.0}{1}} = 4005 \text{ ft/min}$$

Notice that as the density factor goes down, the air velocity increases.

8.4 DallaValle Equation

The following form of the DallaValle equation calculates the capture velocity required for a plain opening hood (no flange):

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$$V = \frac{Q}{10x^2 + A} \tag{181}$$

where

V = capture velocity at distance x from the hood opening, ft/min

Q = air flow, ft^3/min

x = centerline distance from hood opening to target area, ft

A = area of the hood opening, ft^2

Important: The DallaValle equation is valid when *x* is not greater than 1.5 times the equivalent diameter of the hood opening. Using the equation outside this limitation will result in erroneous answers. Many equations used in industrial hygiene and safety have such limitations, so always verify the limitations of any equation or model before applying it.

Problem: What volumetric flow rate is required in a 6 inch round plain duct hood located 9 inches from a location requiring a capture velocity of 100 fpm?

Solution: First, the area of the hood is required in ft^2:

$$A = \frac{\pi d^2}{4} = \frac{\pi (6/12)^2}{4} = 0.196 \text{ ft}^2$$

Next, re-arranging equation (181) and substituting the appropriate values provides:

$$Q = V\left(10x^2 + A\right) = 100\left(10\left(\frac{9}{12}\right)^2 + 0.196\right) = 582 \text{ cfm}$$

8.5 Hood Static Pressure

The hood static pressure equation can be used to calculate the hood static pressure required to overcome losses as air enters a hood.

$$\left|SP_h\right| = VP_d + h_e \tag{182}$$

where

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SP_h = value of hood static pressure, in. wc

VP_d = velocity pressure in duct, in. wc

h_e = hood entry loss, in. wc

The hood entry loss (h_e) can be defined as:

$$h_e = F_h \cdot VP_d \tag{183}$$

where

F_h = hood entry loss factor, dimensionless

Values for F_h vary depending on hood entry design with typical values ranging from 0.04 to 0.93.

Problem: Calculate the hood static pressure when the duct velocity pressure is 1.25 in. wc and the hood entry loss is 0.9 in. wc.

Solution:

$$|SP_h| = VP_d + h_e = 1.25 \text{ in.wc} + 0.9 \text{ in.wc} = |2.15 \text{ in.wc}|$$

Note: Here the static pressure is calculated as an absolute value. Since this is a hood static pressure, SP_h = 2.15 in. wc.

Problem: Based on the same data, what is the hood entry loss factor for this hood?

Solution: Re-arranging equation (183) and substituting leads to:

$$F_h = \frac{VP_d}{h_e} = \frac{1.25 \text{ in.wc}}{0.9 \text{ in.wc}} = 1.39$$

8.6 Hood Entry Coefficient and Loss

Hoods are not perfect at turning available static pressure into velocity pressure. As a result the actual flow entering a hood is related to the theoretical maximum flow by the hood entry coefficient, C_e.

$$C_e = \sqrt{\frac{VP_d}{|SP_h|}} \tag{184}$$

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where

C_e = hood entry coefficient, dimensionless

The hood entry coefficient can be used to determine the hood entry losses for a given velocity pressure,

$$h_e = \frac{\left(1 - C_e^2\right)}{C_e^2} VP_d \tag{185}$$

Problem: Based on the data provided and calculated in the previous sample problem, determine the hood entry coefficient.

Solution: The duct velocity pressure was 1.25 in. wc and the hood entry loss is 0.9 in. wc. The hood static pressure was found to be 2.15 in. wc.

$$C_e = \sqrt{\frac{VP_d}{|SP_h|}} = \sqrt{\frac{1.25}{|2.15|}} = 0.76$$

Problem: Using the calculated hood entry coefficient, verify the hood entry loss factor.

Solution:

$$h_e = \frac{\left(1 - C_e^2\right)}{C_e^2} VP_d = \frac{\left(1 - 0.76^2\right)}{0.76^2} 1.25 \text{ in.wc} = 0.9$$

As expected, the values are equal.

Note that equations (183) and (185) can be combined to demonstrate:

$$F_h = \frac{\left(1 - C_e^2\right)}{C_e^2} \tag{186}$$

Equation (186) can then be rearranged as follows:

$$F_h = \frac{1}{C_e^2} - 1 \tag{187}$$

$$F_h + 1 = \frac{1}{C_e^2} \tag{188}$$

now solving for C_e

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$$C_e = \sqrt{\frac{1}{F_h + 1}} \tag{189}$$

Problem: Given the hood entry loss factor of 0.9 in. wc used in the previous example, calculate the hood entry loss coefficient and compare the result to that answer found using equation (184).

Solution: The value calculated using equation (184) was 0.76. For equation (189) we find:

$$C_e = \sqrt{\frac{1}{0.9 + 1}} = 0.73$$

The difference (~4%) is due to the precision of the values carried through the equations; that is with each rounding comes a loss of precision.

8.7 Converging Duct Flows and Losses

Another type of loss encountered with ventilation flows occurs when two ducts merge and turbulence causes losses. This can be calculated as follows:

$$VP_r = \left(\frac{Q_1}{Q_3}\right) VP_1 + \left(\frac{Q_2}{Q_3}\right) VP_2 \tag{190}$$

where

VP_r = resulting velocity pressure of the merged flows, in. wc

Q_3 = volumetric flow rate of the merged flows, ft^3/min

Q_1 = volumetric flow rate of duct 1, ft^3/min

VP_1 = velocity pressure in duct 1, in. wc

Q_2 = volumetric flow rate of duct 2, ft^3/min

VP_2 = velocity pressure in duct 2, in. wc

Problem: Two ventilation branch ducts converge. The first has a volumetric flow rate of 1500 cfm at a velocity pressure of 1.25 in. wc. The second has a volumetric flow rate of 2000 cfm at a velocity pressure of 0.75 in. wc. Calculate the resulting volumetric flow and velocity pressure.

Solution: First, the volumetric flow rate is simply the sum of the two flows, or 3500 cfm.

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The resulting velocity pressure is calculated using equation (190).

$$VP_r = \left(\frac{Q_1}{Q_3}\right)VP_1 + \left(\frac{Q_2}{Q_3}\right)VP_2 = \left(\frac{1500}{3500}\right)1.25 + \left(\frac{2000}{3500}\right)0.75 = 0.96 \text{ in.wc}$$

Another consideration of two ducts joining is the resulting flow and static pressure. In effect, this can be used to balance static pressures during the design of a ventilation system by determining a new volumetric flow for one duct based on the governing static pressure. This can be written as

$$Q_{cor} = Q_{design}\sqrt{\frac{SP_{gov}}{SP_{duct}}} \qquad (191)$$

where

Q_{cor} = corrected (new) flow rate, ft^3/min

Q_{design} = design (existing) flow rate, ft^3/min

SP_{gov} = governing static pressure, in. wc

SP_{duct} = design static pressure, in. wc

The equation can also be used to determine a new volumetric flow rate in a duct when an old flow and static pressure are known and a new static pressure is measured.

Problem: Two ventilation branch ducts converge. Preliminary design calculations show the following: The first has a volumetric flow rate of 2000 cfm and a static pressure of 1.25 in. wc. The second has a volumetric flow rate of 1500 cfm and a static pressure of 1.20 in. wc. Since the static pressures must be equivalent at the junction, calculate a corrected flow for the second branch.

Solution:

$$Q_{cor} = Q_{design}\sqrt{\frac{SP_{gov}}{SP_{duct}}} = 1500\sqrt{\frac{1.25}{1.20}} = 1531 \text{ cfm}$$

Therefore, a flow of 1531 cfm in the second branch will result in the pressures at the junction being balanced (which is required).

Note: This approach of balancing converging duct flows is only appropriate for small differences in static pressure (i.e., about 20%).

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8.8 Further Applications of Flow and Velocity Equations

The various volumetric flow rate and velocity equations and corrections derived above can be combined and written in a variety of useful formats. For example, equation (170):

$$V = 4005\sqrt{VP_d} \tag{192}$$

and equation (184):

$$C_e = \sqrt{\frac{VP_d}{|SP_h|}} \tag{193}$$

can be combined to yield:

$$V = 4005C_e\sqrt{|SP_h|} \tag{194}$$

Problem: Calculate the velocity in an 8-inch round duct from a hood if the hood static pressure measurement is 2.0 in. wc and the hood entry coefficient is 0.72 (round duct, plain end).

Solution:

$$V = 4005C_e\sqrt{|SP_h|} = 4005(0.72)\sqrt{|2.0|} = 4078 \text{ fpm}$$

Recalling equation (154)

$$Q = A \cdot V \tag{195}$$

Combing equations (194) and (195) leads to

$$Q = 4005C_eA\sqrt{|SP_h|} \tag{196}$$

Problem: Calculate the volumetric flow rate in an 8-inch round duct from a hood if the hood static pressure measurement is 2.0 in. wc and the hood entry coefficient is 0.72 (round duct, plain end).

Solution:

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$$Q = 4005 C_e A \sqrt{|SP_h|} = 4005(0.72)\left[\frac{\pi}{4}\left(\frac{8}{12}\right)^2\right]\sqrt{|2.0|} = 1423 \text{ cfm}$$

We can then modify this by recalling from above

$$C_e = \sqrt{\frac{1}{F_h + 1}} \tag{197}$$

and including the density correction factor (equation (172)) results in

$$Q = 4005A\sqrt{\frac{SP_h}{df\,(1+F_h)}} \tag{198}$$

Problem: Calculate the volumetric flow rate in an 8-inch round flanged hood if the static pressure is 0.8 in. wc., the hood entry loss factor is 0.50 and the duct is moving air at standard atmospheric pressure and 95 ^{o}F.

Solution: First, we need to use equation (172) to determine the density factor:

$$df = \left(\frac{530}{T+460}\right)\cdot\left(\frac{BP}{29.92}\right) = \left(\frac{530}{95+460}\right)\cdot\left(\frac{29.92}{29.92}\right) = 0.95$$

Next we use equation (198):

$$Q = 4005A\sqrt{\frac{SP_h}{df\,(1+F_h)}} = 4005\left[\frac{\pi}{4}\left(\frac{8}{12}\right)^2\right]\sqrt{\frac{2.0}{0.95(1+0.5)}} = 1656 \text{ cfm}$$

A similar substitution (and using equation (169) from above) results in

$$Q = 1096A\sqrt{\frac{SP_h}{\rho(1+F_h)}} \tag{199}$$

Note here the density correction factor is not needed because this form of the equation requires the density of the air at the appropriate temperature and pressure.

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Problem: Using equation (199), calculate the volumetric flow rate in an 8-inch round flanged hood if the static pressure is 0.8 in. wc., the hood entry loss factor is 0.50, and the duct is moving air at standard atmospheric pressure and 95 ^{o}F.

Solution: First, we need the density of air at 95 ^{o}F. There are various sources that can be consulted. One simple way that only requires the density at standard temperature and pressure (STP) is the relationship:

$$\rho T = \text{constant}$$

Knowing the density of air at STP (0.075 lb/ft^3) leads to:

$$\rho_{95} = \frac{\rho_{STP} T_{STP}}{T_{95}} = \frac{(0.075 \text{ lbs/ft}^3)(460 + 68\ ^{o}\text{F})}{460 + 95\ ^{o}\text{F}} = 0.071 \text{ lbs/ft}^3$$

Next, substituting values into equation (199) leads to:

$$Q = 1096 A \sqrt{\frac{SP_h}{\rho(1+F_h)}} = 1096\left[\frac{\pi}{4}\left(\frac{8}{12}\right)^2\right]\sqrt{\frac{2}{0.071(1+0.5)}} = 1658 \text{ cfm}$$

As expected, this value is very close to the 1656 cfm calculated in the previous sample problem. The small difference is due to rounding in the equations.

Note that equation (199) assumes standard pressure, which was also used (but not required) in the previous problem.

8.9 Dilution Ventilation

Dilution ventilation is an important aspect of airborne contaminant control. The concentration of a gas or vapor as a function of time can be derived from a differential material balance which, when integrated, relates the ventilation to the generation and removal of a contaminant.

This material balance can be written as

$$VdC = Gdt - Q'Cdt \tag{200}$$

where

V = volume of enclosure

C = concentration of gas or vapor at time t

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G = rate of generation of contaminant

Q = rate of ventilation

K = mixing factor

$Q' = Q/K$ = effective rate of ventilation

From equation (200), several useful relationships can be derived. Rearranging equation (200) and integrating leads to:

$$\int_{C_1}^{C_2} \frac{dC}{G - Q'C} = \frac{1}{V}\int_{t_1}^{t_2} dt \tag{201}$$

Recall that (for a definite integral)

$$\int_a^b \frac{dx}{x} = \ln|x| \tag{202}$$

and

$$\ln a - \ln b = \ln\left(\frac{a}{b}\right) \tag{203}$$

So equation (201) becomes

$$\ln\left(\frac{G - Q'C_2}{G - Q'C_1}\right) = -\frac{Q'}{V}\left(t_2 - t_1\right) \tag{204}$$

where

ln = natural logarithm

V = volume of enclosure, ft^3

C_1 = initial concentration of gas or vapor, parts-per-million/10^6, ppm/10^6

C_2 = final concentration of gas or vapor, ppm/10^6

G = rate of generation of contaminant, ft^3/min

$Q' = Q/K$ = effective rate of ventilation, ft^3/min

where Q = rate of ventilation, ft^3/min

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K = mixing factor (typical values range from 1 to 10), nondimmensional

t_2 = final time, min

t_1= initial time, min

Problem: Acetone evolves at a rate of 3.5 cfm in a room that measures 30' x 50' x 12'. If an initial concentration is measured at 25 ppm, what will the concentration be after 15 minutes of 3000 cfm of dilution air? Assume K=1 (i.e., $Q' = Q$).

Solution: We can use equation (204), but the final concentration (C_2) is embedded in this form of the equation, so we must solve for C_2.

$$\ln\left(\frac{G-Q'C_2}{G-Q'C_1}\right)=-\frac{Q'}{V}\left(t_2-t_1\right)$$

$$\ln\left(\frac{3.5-3000\cdot C_2}{3.5-3000\left(0.000025\right)}\right)=-\frac{3000}{18000}\left(15-0\right)$$

$$\frac{3.5-3000\cdot C_2}{3.5-3000\left(0.000025\right)}=e^{\left(-\frac{3000}{18000}(15-0)\right)}$$

$$\frac{3.5-3000\cdot C_2}{3.425}=0.0821$$

$$C_2=\frac{\left(0.0821\right)\left(3.425\right)-3.5}{-3000}=0.00107=1073\text{ ppm}$$

It is important to note that the contaminant concentration, if given in ppm, must be converted to a volume fraction. This can be done by the following equation which relates concentrations in ppm to volumetric fractions for airborne gases and vapors.

$$ppm_{contam}=\frac{V_{contam}}{V_{air}}x10^6 \qquad (205)$$

This can be written as

$$\frac{ppm_{contam}}{10^6}=\frac{V_{contam}}{V_{air}} \qquad (206)$$

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Problem: What is the volume of airborne acetone after 15 minutes in the sample problem above?

Solution: From the above equation, we know at 15 minutes the acetone is at 1073 ppm. We also know the volume of the enclosure is 18,000 ft^3. Therefore, we can re-arrange equation (206) as follows:

$$V_{contam} = V_{air}\left(\frac{ppm_{contam}}{10^6}\right) = 18,000 \text{ ft}^3\left(\frac{1073}{10^6}\right) = 19.3 \text{ ft}^3$$

From the general form dilution equation (204) comes other dilution equations that address special cases. For example, if we assume at time $t=0$, the concentration is $C_1=0$, then equation (204) is simplified and becomes:

$$\ln\left(\frac{G-Q'C}{G}\right) = -\frac{Q'}{V}t \tag{207}$$

and since

$$e^{\ln(x)} = x \tag{208}$$

Equation (207) can be written as

$$\frac{G-Q'C}{G} = e^{-\frac{Q'}{V}t} \tag{209}$$

Problem: Acetone begins to be evolved at a rate of 3.5 cfm in a room that measures 30' x 50' x 12' high. If the initial acetone concentration is 0 ppm, what will the concentration be after 15 minutes of 3000 cfm of dilution air? Assume K=1.

Solution: This is an application of equation (209).

$$\frac{G-Q'C}{G} = e^{-\frac{Q'}{V}t}$$

$$C = \frac{G\cdot e^{-\frac{Q'}{V}t} - G}{-Q'}$$

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$$C = \frac{3.5 \cdot e^{-\frac{3000}{18000}15} - 3.5}{-3000} = 0.001071 = 1071 \text{ ppm}$$

Now consider the case where a volume of air is contaminated at some initial concentration and we wish to calculate the change in concentration over time due to dilution ventilation when there is no new contaminant being added (i.e., $G = 0$). For this we start with the material balance of:

$$VdC = -Q'Cdt \tag{210}$$

Similar to above, we can find:

$$\int_{C_1}^{C_2} \frac{dC}{C} = -\frac{Q'}{V} \int_{t_1}^{t_2} dt \tag{211}$$

Integration leads to

$$\ln\left(\frac{C_2}{C_1}\right) = -\frac{Q'}{V}\left(t_2 - t_1\right) \tag{212}$$

and this equation can be rearranged to yield

$$t_2 - t_1 = -\frac{V}{Q'} \ln\left(\frac{C_2}{C_1}\right) \tag{213}$$

Problem: Acetone is used in a room that measures 30' x 50' x 12'. An initial concentration is measured at 5000 ppm, and the acetone use is stopped (i.e., no more acetone vapors evolve). With 3000 cfm of dilution air, how long would it take to reach a level of 250 ppm? Assume K=1.

Solution:

$$t_2 - t_1 = -\frac{V}{Q'} \ln\left(\frac{C_2}{C_1}\right) = -\frac{18000}{3000} \ln\left(\frac{250}{5000}\right) = 18 \min$$

Fractions in Equations Note that in the above equations, the fraction C_2/C_1 appears. In this case, we do not worry about units as they cancel to form a dimensionless fraction; they only need to have the same units. This simplifies this

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problem.

Now consider the case in which we seek to identify the constant level of an airborne contaminant when the generation rate and ventilation rate are known and there is a steady concentration of contaminant in the supply air.

$$C = \left(\frac{G}{Q^{'}} x10^{6} \right) + C_{supply} \qquad (214)$$

where

the variables and units are as defined above, and

C_{supply} = concentration of contaminant in the supply air, ppm

Problem: Connected rooms utilize a cascading ventilation system where air with lower contamination levels moves towards rooms with higher concentrations before reaching filters. Assume a room with a toluene process that evolves 0.5 cfm of toluene is supplied by 2500 cfm or air coming from a room with an airborne concentration that is limited to 50 ppm. Determine the steady-state concentration of toluene in the room.

Solution:

$$C = \left(\frac{G}{Q^{'}} x10^{6} \right) + C_{supply} = \left(\frac{0.5}{2500} x10^{6} \right) + 50 \text{ ppm} = 250 \text{ ppm}$$

Notice that the room volume is not required.

8.10 Room Air Changes per Hour

A common value for indoor air ventilation is the number of air changes per hour (ACH). Building and mechanical codes typically specify minimum ACH for most occupancy types. The ACH can easily be calculated as follows:

$$N_{changes} = \frac{60Q}{V_{room}} \qquad (215)$$

where

$N_{changes}$ = number of air changes per hour (ACH)

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60 = conversion factor for minutes to hours, min/hour

Q = room ventilation rate, ft^3/min

V_{room} = room volume, ft^3

Problem: A room measures 30' x 50' x 12' high. How many air changes per hour (ACH) are required to provide a ventilation rate of 3000 cfm?

Solution:

$$N_{changes} = \frac{60Q}{V_{room}} = \frac{60 \cdot 3000}{18,000} = 10 \text{ ACH}$$

8.10.1 Dilution Ventilation Based on Room Air Changes

When a room starts with no concentration of an airborne contaminant, but a contaminant is added at a steady rate over time, the time-dependant concentration can be calculated based on the number of air changes per hour. This is shown here:

$$C = \frac{G}{Q'}\left(1 - e^{-Nt/60}\right)x10^6 \tag{216}$$

where

C = concentration at time t, ppm/10^6

G = rate of generation of contaminant, ft^3/min

Q' = effective rate of ventilation, ft^3/min

e = natural logarithm, 2.71828…

t = time elapsed, hours

N = number of air changes per hour

60 = conversion from minutes to hours

Problem: Starting with equation (209) and (215), derive equation (216).

Solution: First, we must re-arrange equation (209):

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$$\frac{G - Q'C}{G} = e^{-\frac{Q'}{V}t}$$

$$C_{ppm} = \frac{G \cdot e^{-\frac{Q'}{V}t} - G}{-Q'} \text{ x } 10^6$$

$$C_{ppm} = \frac{G \cdot e^{-\frac{Q'}{V}t} - G}{-Q'} \text{ x } 10^6 = \frac{G}{Q'}\left(1 - e^{-\frac{Q'}{V}t}\right) \text{ x } 10^6$$

Next, equation (215) can be re-arranged as follows:

$$\frac{N_{changes}}{60} = \frac{Q}{V_{room}}$$

Substituting this into the preceding equation leads to:

$$C_{ppm} = \frac{G}{Q'}\left(1 - e^{-Nt/60}\right) x 10^6$$

When a room starts with a known concentration of an airborne contaminant, and no additional contaminate is added, the time-dependant concentration (dilution) can be calculated based on the number of air changes per hour. This is shown here:

$$C = C_0 e^{-tN} \tag{217}$$

where

C = concentration at time t, units to match C_0

C_0 = initial concentration, units to match C

t = time, hours

N = number of air changes per hour

Problem: A process area has a ventilation system that provides 20 ACH. What is the concentration of an airborne contaminant after 15 minutes if the initial concentration is

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300 ppm?

Solution:

$$C = C_0 e^{-tN} = \left(300\text{ ppm}\right) e^{-\left(0.25\text{ hr}\right)\left(20\text{ ACH}\right)} = 2.02\text{ ppm}$$

8.11 Dilution to Control Evaporation

The following equation can be used to calculate the ventilation required to keep an evaporating contaminant (e.g., a solvent) below a desired concentration. The concentration can be a TLV, LFL (LEL) or any other desired concentration. Note that this equation is based on pints/min of evaporating contaminant.

$$Q = \frac{\left(403\right)\left(SG\right)\left(ER\right)\left(K\right)\left(10^6\right)}{\left(MW\right)\left(C\right)} \tag{218}$$

where

Q = volumetric flow required to limit concentration, ft^3/min

403 = constant for units used

SG = specific gravity, nondimensional

ER = evaporation rate, pints/min

K = ventilation (dilution) safety factor, nondimensional

10^6 = unit conversion (ppm to volume percent)

MW = molecular weight, g

C = contaminant concentration in air, ppm

Problem: Acetone evaporates at a rate of 0.1 pints/min. How much dilution air is required to maintain the concentration below the TLV? Assume the TLV is 500 ppm and a ventilation safety factor of 5. The molecular weight is 58.08, and the specific gravity is 0.79.

Solution:

$$Q = \frac{\left(403\right)\left(SG\right)\left(ER\right)\left(K\right)\left(10^6\right)}{\left(MW\right)\left(C\right)} = \frac{\left(403\right)\left(0.79\right)\left(0.1\right)\left(5\right)\left(10^6\right)}{\left(58.08\right)\left(500\right)} = 5482\text{ cfm}$$

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8.12 Fan Laws and Equations

Many engineered controls for airborne contaminant control require the use of fans.

Two equations that describe a fan's ratings are fan static pressure and fan total pressure. These are shown here.

$$FSP = SP_{out} - SP_{in} - VP_{in} \tag{219}$$

where

FSP = fan static pressure; this can also be shown as SP_{fan}, in. wc

SP_{out} = static pressure out; measured on the outlet side of the fan, in. wc

SP_{in} = static pressure in; measured on the inlet side of the fan, in. wc

VP_{in} = velocity pressure on the inlet side of the fan, in. wc

Problem: Calculate the fan static pressure if the static pressure on the inlet side is -2.5 in. wc, the static pressure on the outlet side is 0.75 in. wc, and the velocity pressure is 1 in. wc.

Solution:

$$FSP = SP_{out} - SP_{in} - VP_{in} = (0.75 \text{ in.wc}) - (-2.5 \text{ in.wc}) - (1 \text{ in.wc}) = 2.25 \text{ in.wc}$$

Important: The static pressure on the inlet side is always negatively signed. The static pressure on the outlet side and the velocity pressure is always positively signed.

The fan total pressure is defined as:

$$FTP = TP_{out} - TP_{in} \tag{220}$$

where

FTP = fan total pressure, in. wc

TP_{out} = total pressure measured at the outlet, in. wc

TP_{in} = total pressure measured at the inlet, in. wc

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Problem: A fan supplies air at a velocity of 4000 fpm. At the same fan, the inlet and outlet static pressures are -5.0 in. wc and 0.6 in. wc, respectively. Determine the fan total pressure. Assume the inlet and outlet velocity pressures are 1.0 in. wc and 0.7 in. wc, respectively.

Solution: From equation (160) we know:

$$TP = VP + SP$$

Therefore, equation (220) can be written:

$$FTP = TP_{out} - TP_{in} = \left(VP_{out} + SP_{out}\right) - \left(VP_{in} + SP_{in}\right)$$

$$FTP = \left(0.7 + 0.6\right) - \left(1.0 + -5.0\right) = 5.3 \text{ in.wc}$$

Important: The static pressure on the inlet side is always negatively signed. The static pressure on the outlet side and the velocity pressure is always positively signed.

8.12.1 Fan Laws

The following three equations are known as the fan laws; they are also referred to as affinity laws. Notice all the "laws" are a function of size and speed (revolutions per minute). Sometimes these equations are written without showing the "size" term. When this is done, this assumes the fan size cannot be changed; such as after a fan is installed. Note that these equations apply to a "family" of fans of similar design and manufacturer. They may not be applied to a mix of various designs. Also note the various powers used in the fan laws.

The first equation relates the volumetric movement of a fan to size (to the third power) and speed.

$$Q_2 = Q_1 \left(\frac{Size_2}{Size_1}\right)^3 \left(\frac{RPM_2}{RPM_1}\right) \tag{221}$$

where

Q_2 = volumetric flow rate for condition 2, ft^3/min

Q_1 = volumetric flow rate for condition 1, ft^3/min

$Size_2$ = fan diameter for condition 2, inches

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$Size_1$ = fan diameter for condition 1, inches

RPM_2 = fan speed for condition 2, rpm

RPM_1 = fan speed for condition 1, rpm

Problem: A fan with a 6 inch impeller operates at 2000 RPM to supply 1500 cfm. If the impeller size and speed is changed to 8 inches and 2500 RPM, what will be the new flow?

Solution:

$$Q_2 = Q_1\left(\frac{Size_2}{Size_1}\right)^3\left(\frac{RPM_2}{RPM_1}\right) = 1500\left(\frac{8}{6}\right)^3\left(\frac{2500}{2000}\right) = 4444 \text{ cfm}$$

The second equation relates the fan pressure to size and speed (both to the second power).

$$P_2 = P_1\left(\frac{Size_2}{Size_1}\right)^2\left(\frac{RPM_2}{RPM_1}\right)^2 \qquad (222)$$

where

variables and units are as defined above

P_2 = system pressure for condition 2, in. wc

P_1 = system pressure for condition 1, in. wc

Problem: If a fan size remains the same, how much faster would the fan have to turn to increase the pressure 50%?

Solution: Since the fan size does not change, that portion of the equation equals 1.0. Equation (222) can then be re-arranged to solve for RPM_2.

$$P_2 = P_1\left(\frac{RPM_2}{RPM_1}\right)^2$$

$$RPM_2 = RPM_1\sqrt{\frac{P_2}{P_1}}$$

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$$RPM_2 = RPM_1\sqrt{\frac{P_2}{P_1}} = 1\sqrt{\frac{1.5}{1}} = 1.22$$

Therefore, a 22% increase in RPM will increase the pressure by 50 percent. Also, note we did not use specific values for the speed and pressure, only multipliers since we were only looking for a multiplier.

The third equation relates a fan's power requirement to size (to the fifth power) and speed (to the third power).

$$PWR_2 = PWR_1\left(\frac{Size_2}{Size_1}\right)^5\left(\frac{RPM_2}{RPM_1}\right)^3 \qquad (223)$$

where

variables and units are as defined above

PWR_2 = fan horsepower for condition 2, horsepower, hp

PWR_1 = fan horsepower for condition 1, hp

Problem: An 8 inch fan operates at 2500 RPM with a breaking horsepower (BHP) of 30. The fan size is decreased to 6 inches and the speed increased to 3000 RPM. What is the new BHP?

Solution:

$$PWR_2 = PWR_1\left(\frac{Size_2}{Size_1}\right)^5\left(\frac{RPM_2}{RPM_1}\right)^3 = 30\left(\frac{6}{8}\right)^5\left(\frac{3000}{2500}\right)^3 = 12.3 \text{ BHP}$$

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Sound and Noise

9 Sound and Noise

9.1 Sound Intensity

Safety and industrial hygiene professionals typically deal with sound pressure and not sound intensity. Generally, there is no direct relationship between sound pressure and sound intensity. However, for a plane wave there is a relationship. This relationship can be used in a free field at a distance from the source.

$$I = \frac{p^2}{\rho c} \tag{224}$$

where

I = sound intensity, W/m^2

p = rms sound pressure, Pa

ρ = density of air, kg/m^3

c = speed of sound in air, m/sec

Note: rms stands for *root mean square*, and the value ρc is the characteristic specific acoustic impedance and is equal to 413 $N\text{-}s/m^3$ for air at 20 ^oC.

Problem: Calculate the sound intensity of a 0.2 Pa source. Assume the air temperature is 20 ^oC.

Solution:

$$I = \frac{p^2}{\rho c} = \frac{(0.2 \text{ Pa})^2}{413 \text{ N-s/m}^3} = 9.7\text{x}10^{-5} \text{ W/m}^2$$

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9.2 Sound Pressure Level (SPL)

Sound pressure level (SPL) or sound level is a logarithmic measure of the effective sound pressure of a sound relative to a reference value. It is measured in decibels (dB) above a standard reference level, typically 20 µPa RMS (which is usually considered the threshold of human hearing at 1 kHz). Mathematically, sound pressure level (SPL) can be written as:

$$SPL = 20\left(\log \frac{P}{P_0}\right) \tag{225}$$

where

SPL = sound pressure level, dB

P = measured rms sound pressure, Pa

P_0 = reference rms sound pressure, Pa (P_o is typically 20 µPa)

Problem: Calculate the sound pressure level (in dB) due to a sound pressure of 0.5 Pa.

Solution:

$$SPL = 20\left(\log \frac{P}{P_0}\right) = 20\left(\log \frac{0.5 \text{ Pa}}{20\text{x}10^{-6} \text{ Pa}}\right) = 88 \text{ dB}$$

Sound pressure level (SPL) can be related to the sound intensity (power) by:

$$SPL = 10\left(\log \frac{I}{I_0}\right) \tag{226}$$

where

SPL = sound pressure level, dB

I = sound intensity, W/m^2

I_0 = reference sound intensity, W/m^2 (I_0 is typically 10^{-12} W/m^2)

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Problem: Calculate the sound pressure level for a measured intensity of 0.005 W/m^2.

Solution:

$$SPL = 10\left(\log\frac{I}{I_0}\right) = 10\left(\log\frac{0.005\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = 97\ \text{dB}$$

Sound pressure level decreases over distance. The change is not linear, rather it changes logarithmically as follows:

$$SPL_2 = SPL_1 + 20\log\left(\frac{d_1}{d_2}\right) \tag{227}$$

where

SPL_2 = sound pressure level at distance d_2, dB

SPL_1 = sound pressure level at distance d_1 dB

d_1 = distance where SPL_1 was measured

d_2 = distance where SPL_2 was measured

Problem: Sound measurements record an average sound pressure level of 85 dB at a location 10 feet away from a punch-press. What is the expected sound pressure level at 15 feet from the press?

Solution:

$$SPL_2 = SPL_1 + 20\log\left(\frac{d_1}{d_2}\right) = 85\ \text{dB} + 20\log\left(\frac{10\ \text{ft}}{15\ \text{ft}}\right) = 81.5\ \text{dB}$$

Reminder: Since the distances are in a fraction, any units can be used as long as they are consistent.

9.2.1 Addition of Sound Pressure Levels (SPLs)

Due to their logarithmic nature, sound pressure levels cannot simply be added together, rather they must be added while accounting for their logarithmic nature.

The two following equations can be used to add sound pressure levels.

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$$SPL_{total} = 10\log\sum_{i=1}^{N}10^{\frac{SPL_i}{10}} \tag{228}$$

where

SPL_{total} = total (sum) of all sound pressure levels, dB

i = count of individual sound pressure levels

N = number of individual sound pressure levels summed

SPL_i = SPL of sound i, dB

Problem: Calculate the combined sound pressure level created by three sources measured at 80 dB, 95 dB and 90 dB.

Solution:

$$SPL_{total} = 10\log\left(10^{\frac{80}{10}} + 10^{\frac{95}{10}} + 10^{\frac{90}{10}}\right) = 96.3 \text{ dB}$$

A simple form of the equation can be derived for cases involving a number of identical sources.

$$SPL_{total} = SPL_i + 10\log(n) \tag{229}$$

where

SPL_{total} = total (sum) of all sound pressure levels, dB

SPL_i = SPL of a single source, dB

n = number of identical sound pressure levels summed

Problem: Four machines are to be collocated. Produce literature indicates an expected sound pressure level of 80 dB (at a reference distance) for each machine. What is the expected combined sound pressure level?

Solution:

$$SPL_{total} = SPL_i + 10\log(n) = 85 + 10\log(4) = 85 + 6 = 91 \text{ dB}$$

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Note: Equation (228) can also be used as follows:

$$SPL_{total} = 10\log\left(4\left(10^{\frac{85}{10}}\right)\right) = 91\text{dB}$$

Another form that you may see for adding sound pressure levels (mathematically identical to equation (228)) is:

$$L_{PT} = 10\log\left(\sum_{i=1}^{N} 10^{\frac{L_{Pi}}{10}}\right) \tag{230}$$

where

L_{PT} = total (sum) of all sound pressure levels, dB

i = count of individual sound pressure levels

N = number of individual sound pressure levels summed

Lp_i = SPL of sound i, dB

This equation can be used for any number of varying sources. When you have two sources, the equation simplifies to the following:

$$L_{Total} = L_1 + 10\log\left(10^{\frac{L_2 - L_1}{10}} + 1\right) \tag{231}$$

where

L_{Total} = total (sum) of two source sound pressure levels, dB

L_1 = SPL of sound I, dB

L_2 = SPL of sound 2, dB

Problem: Two machines are to be collocated. Produce literature indicates an expected sound pressure level of 80 dB and 85 dB (at a reference distance) for the machines. What is the expected combined sound pressure level?

Solution:

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$$L_{Total} = L_1 + 10\log\left(10^{\frac{L_2 - L_1}{10}} + 1\right) = 80 + 10\log\left(10^{\frac{85-80}{10}} + 1\right) = 86.2 \text{ dB}$$

Note: The assignment of the higher or lower sound to L_1 is not required. Try reversing the values for L_1 and L_2 and check the solution.

9.2.2 Time Weighted Equivalent Sound Pressure Level

Sometimes you wish to determine an equivalent sound pressure level for a variety of sounds (noises) experienced over varying durations. This can be found as follows:

$$L_{eq} = 10\log\frac{1}{T}\left(\sum_{i=1}^{N}\left(10^{\frac{L_i}{10}} t_i\right)\right) \tag{232}$$

where

L_{eq} = time weighed equivalent sound pressure level, dB

T = total observation time of the sounds, hours

i = count of individual sound pressure levels

N = number of individual sound pressure levels summed

L_i = SPL of sound i, dB

t_i = duration of sound i, hours

Note that each sound exposure is multiplied by its duration, and then the total duration is divided out to yield the weighted average.

Problem: Calculate the equivalent sound pressure level for the following measurements: 80dB for 2 hours, 92 dB for 1 hour, 94 dB for 2 hours, and 80 dB for 3 hours.

Solution:

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$$L_{eq} = 10\log\frac{1}{T}\left(\sum_{i=1}^{N}\left(10^{\frac{L_i}{10}}t_i\right)\right)$$

$$L_{eq} = 10\log\frac{1}{T}\left(\sum_{i=1}^{N}\left(10^{\frac{80}{10}}\cdot 2+10^{\frac{92}{10}}\cdot 1+10^{\frac{94}{10}}\cdot 2+10^{\frac{80}{10}}\cdot 3\right)\right) = 89.6\text{ dB}$$

9.3 Sound Power Level

The sound power level (L_W) of a signal with sound power W (watts) is:

$$L_W = 10\log\left(\frac{W}{W_0}\right) \tag{233}$$

where

L_w = sound power level, dB

W = sound power, W

W_0 = reference sound intensity, W (W_0 is commonly set to 10^{-12} W)

Problem: A sound system produces 50 Watts of power. What is the sound power level?

Solution:

$$L_W = 10\log\left(\frac{W}{W_0}\right) = 10\log\left(\frac{50\text{ W}}{10^{-12}\text{ W}}\right) = 137\text{ dB}$$

Within a free field, the sound pressure level and sound power level can be related by the following equation:

$$L_p = L_w - 20\log r - 0.5 + DI + T \tag{234}$$

where

L_p = sound pressure level, dB

L_w = sound power level, dB

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r = distance, ft

0.5 = a constant for English units

DI = direction index (see below), dB

T = temperature and pressure correction factor (ignored at standard conditions), dB

and

$$DI = 10\log Q \tag{235}$$

where

DI = directivity index, dB

Q = directivity factor, nondimensional

Q = 1 for spherical radiation

2 for ½ spherical radiation

4 for ¼ spherical radiation

8 for 1/8 spherical radiation

Problem: Assume the sound system from the previous sample problem is measured in a free field; calculate the sound pressure level at 15 feet. Assume standard conditions and ½ spherical radiation.

Solution: First, the directivity index must be calculated using equation (235):

$$DI = 10\log Q = 10\log(2) = 3 \text{ dB}$$

Then equation (234) can be applied with a temperature and pressure correction factor set to 0.

$$L_p = L_w - 20\log r - 0.5 + DI + T = 137 - 20\log(15) - 0.5 + 3 + 0 = 116 \text{ dB}$$

9.4 Transmission Loss

The sound transmission loss describes the sound reduction due to a sound striking one surface of a barrier (e.g., a wall) and leaving the other side. It is defined as flows:

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$$TL = 10\log\left(\frac{E_i}{E_t}\right) \tag{236}$$

where

TL = transmission loss, dB

E_i = sound power incident on the barrier, W/m^2

E_t = sound power on the opposite side of the barrier, W/m^2

Equation (236) is also written as:

$$TL = 10\log\left(\frac{1}{\tau}\right) \tag{237}$$

where

τ = transmission coefficient, non-dimensional

The transmission coefficient is frequency dependant.

Problem: What is the transmission loss for a 3-3/4" wall constructed of ½" gypsum on metal studs with no insulation? Assume a frequency of 1000 hZ.

Solution: Various sources on sound transmission coefficients are available. For the wall design described, at a frequency of 1000 Hz, the sound transmission coefficient will be about 0.00003.

$$TL = 10\log\left(\frac{1}{\tau}\right) = 10\log\left(\frac{1}{0.00003}\right) = 45 \text{ dB}$$

9.5 Noise Reduction by Absorption

Noise reduction can be reported as a fraction of the amount of noise absorbed in a room before and after treatment for noise reduction. Mathematically this can be written as:

$$dB = 10\log\left(\frac{A_2}{A_1}\right) \tag{238}$$

where

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dB = noise reduction, dB

A_1 = total amount of absorption before treatment, sabins

A_2 = total amount of absorption after treatment, sabins

The Sabin The Sabin is defined as a unit of sound absorption. One square meter of 100% absorbing material has a value of one metric Sabin.

Problem: A plaster ceiling is made of plaster with a sound absorption coefficient of 0.02. The ceiling material is changed to acoustical ceiling tiles with a sound absorption coefficient of 0.6. What is the change in noise reduction for the new material?

Solution: The sound absorption coefficient is typically multiplied by the area to find the sound absorption. Since the area is a constant and equation (238) uses the absorption in fraction (ratio) form, we can simply use the sound absorption coefficient.

$$dB = 10\log\left(\frac{A_2}{A_1}\right) = 10\log\left(\frac{0.6}{0.02}\right) = 14.8 \text{ dB}$$

Note that calculation is only for the change due to the ceiling material. The overall change in the room would have to account for all the surfaces and their absorption coefficients.

9.5.1 Noise Reduction in a Duct

Ducts can be lined to reduce the noise transmission within the duct. This can be expressed as:

$$NR = \frac{12.6P\alpha^{1.4}}{A} \tag{239}$$

where

12.6 = a constant

NR = noise reduction, dB/ft

P = perimeter of duct, in.

α = absorption coefficient of the lining material, nondimensional

A = cross-section area of duct, in^2

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Note that in equation (239) α is raised to the 1.4 power.

Problem: A duct lining material has an absorption coefficient of 0.4. Calculate the reduction in noise as a function of length in a 9" by 24" duct.

Solution: Equation (239) provides the solution in dB/ft, so we can simply substitute values into the equation to find:

$$NR = \frac{12.6P\alpha^{1.4}}{A} = \frac{12.6(2\cdot 9 + 2\cdot 24)(0.4)^{1.4}}{9\cdot 24} = 1.1\text{ dB/ft}$$

9.6 Percent Noise Dose and TWA

The noise dose received over a time period is the summation of the individual noise and duration fractions. Mathematically this can be expressed as:

$$\%D = 100\left(\frac{C_1}{T_1} + \frac{C_2}{T_2} + \ldots + \frac{C_i}{T_i}\right) \qquad (240)$$

where

$\%D$ = noise dose expressed as a percent

C_1 to C_i = exposure duration of each individual noise, hr

T_1 to T_i = corresponding allowable exposure duration of each individual noise, hr

The following is another form of the same equation; mathematically they are identical.

$$\%D = 100\left[\sum_{i=1}^{N} \frac{C_i}{T_i}\right] \qquad (241)$$

Problem: The following sound measurements are made during a work day; 85 dB for 2 hours, 95 dB for 1 hour, 90 dB for 2 hours, 78 dB for 1 hour, 84 dB for 1 hour, 85 dB for 1 hour. Calculate the percent noise dose.

Solution: The values for T_i are calculated using equation (243) below. The following table can be constructed to assists in the calculation:

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dB	C_i	T_i	C_i/T_i
85	2	16.0	0.13
95	1	4.0	0.25
90	2	8.0	0.25
78	1	42.2	0.02
84	1	18.4	0.05
88	1	10.6	0.09

Next, the values for C_i/T_i can be substituted into equation (240) to find:

$$\%D = 100\left(\frac{C_1}{T_1} + \frac{C_2}{T_2} + \ldots + \frac{C_i}{T_i}\right)$$

$$\%D = 100(0.13 + 0.25 + 0.25 + 0.02 + 0.05 + 0.09) = 79\%$$

Values for T_i in the above equations can be calculated as follows:

$$T = \frac{8}{2^{\frac{(L-85)}{3}}} \tag{242}$$

and

$$T = \frac{8}{2^{\frac{(L-90)}{5}}} \tag{243}$$

where

T = allowed exposure time, hr

L = time weighted average (TWA) exposure, dBA

Note that the first exposure calculation is based on the ACGIH TLV for noise of 85 dBA and an exchange rate of 3. The second exposure calculation is based on the OSHA TLV of 90 dBA and an exchange rate of 5.

Problem: Based on OSHA requirements, what is the allowable exposure time for 84 dBA?

Solution: Since we are concerned with OSHA requirements, equation (243) is the appropriate equation to use.

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$$T = \frac{8}{2^{\frac{(L-90)}{5}}} = \frac{8}{2^{\frac{(84-90)}{5}}} = 18.4 \text{ hours}$$

Once the percent dose has been calculated, the equivalent TWA can be calculated as following:

$$TWA = 10 \cdot \log\left(\frac{\%D}{100}\right) + 85\,dBA \tag{244}$$

and

$$TWA = 16.61 \cdot \log\left(\frac{\%D}{100}\right) + 90\,dBA \tag{245}$$

where

TWA = equivalent time weighted average noise exposure, dBA

$\%D$ = noise dose expressed as a percent

Once again, the first TWA calculation is based on the ACGIH TLV for noise of 85 dBA and an exchange rate of 3. The second TWA calculation is based on the OSHA TLV of 90 dBA and an exchange rate or 5.

Problem: Calculate the equivalent time weighted average for a percent noise dose of 79% assuming an OSHA TLV.

Solution: Since we are concerned with OSHA requirements, equation (245) is the appropriate equation to use.

$$TWA = 16.61 \cdot \log\left(\frac{79}{100}\right) + 90\,dBA = 88.3 \text{ dBA}$$

9.7 Frequency by a Fan

The pure tone frequency of a fan can be determined based on the number of fans blades and the rotation speed, as follows:

$$f = \frac{(N)(RPM)}{60} \tag{246}$$

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where

N = number of fan blades

RPM = speed of fan, rpm

60 = time unit conversion

Problem: Determine the fan frequency generated by a fan with 8 blades turning 2400 RPM.

Solution:

$$f = \frac{(N)(RPM)}{60} = \frac{(8)(2400)}{60} = 320 \text{ Hz}$$

9.8 Octave and Third-Octave Bands

Sound frequencies can be complex to assess, so a scale of octave bands and one-third octave bands has been developed to assist in their analyses. Each band covers a specific range of frequencies. The ratio of the frequency of the highest note to the lowest note in an octave is 2:1. The center frequencies for these Octave bands, as defined by ISO, are:

31.5Hz , 63Hz , 125Hz , 250Hz , 500Hz , 1kHz , 2kHz , 4kHz , 8kHz and 16kHz

The ratio of band limits is given by:

$$\frac{f_{n+1}}{f_n} = 2^k \qquad (247)$$

An octave has a center frequency that is $\sqrt{2}$ times the lower cutoff frequency and has an upper cutoff frequency that is twice the lower cutoff frequency. Therefore,

$$f_1 = \frac{f_2}{2} \qquad (248)$$

$$f_2 = 2 f_1 \qquad (249)$$

$$f_c = \sqrt{f_1 \cdot f_2} \qquad (250)$$

$$f_c = \sqrt{2} f_1 \qquad (251)$$

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$$f_c = \frac{f_2}{\sqrt{2}} \tag{252}$$

$$BW = f_2 - f_1 \tag{253}$$

where

f_{n+1} = the upper cutoff frequency

f_n = the lower cutoff frequency

$k = 1$ for full octave bands, and $k = 1/3$ for one-third octave bands.

f_c = the center frequency

BW = bandwidth

Problem: The lower cutoff frequency of an octave band is 354 Hz. Calculate the upper cutoff frequency and the center frequency.

Solution: The upper cutoff frequency is given by equation (249):

$$f_2 = 2f_1 = 2 \cdot 354 = 707 \text{ Hz}$$

The center frequency is given by equation (251):

$$f_c = \sqrt{2} f_1 = \sqrt{2} \cdot 354 \text{ Hz} = 500 \text{ Hz}$$

The center frequency is also given by equation (250):

$$f_c = \sqrt{f_1 \cdot f_2} = \sqrt{354 \cdot 707} = 500 \text{ Hz}$$

Third-Octave bands are calculated the same way, except third-octaves use a one-third power in equation (247). For example,

$$f_2 = \sqrt[3]{2} f_1 \tag{254}$$

Problem: The lower cutoff frequency of a third-octave band is 891 Hz. Calculate the upper cutoff frequency.

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Solution:

$$f_2 = \sqrt[3]{2} f_1 = \sqrt[3]{2} \cdot 891 \text{ Hz} = 1122 \text{ Hz}$$

9.9 Sound Frequency and Wavelength

The frequency and wavelength of a sound are related to the speed of sound in the medium the sound travels through (usually air), and is determined by the following equation:

$$f = \frac{c}{\lambda} \tag{255}$$

where

f = frequency, Hz

c = speed of sound, m/sec

λ = wavelength, m

The speed of sound in air at 20 ^{o}C is 344 m/sec (1125 ft/sec).

Problem: What is the frequency of a sound in air at 20 ^{o}C if the wavelength is 0.75 meters?

Solution:

$$f = \frac{c}{\lambda} = \frac{344 m / \sec}{0.75 m} = 459 \text{ Hz}$$

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Radiation

10 Radiation

10.1 Ionizing

Ionizing radiation results from electromagnetic radiation with sufficient energy to cause the loss of an electron from the matter in which it interacts (i.e., produces ions). The more common ionizing radiation sources encountered in safety and industrial hygiene are alpha particles, beta particles, gamma rays (or photons), X-rays (or photons) and neutrons.

10.1.1 Inverse Square Law

Radiation intensity decreases as a function of distance from its source. The decrease is not linear, rather it is a function of the second power and is defined as:

$$I_2 = I_1\left(\frac{d_1}{d_2}\right)^2 \tag{256}$$

where

I_1 = intensity at distance d_1

I_2 = intensity at distance d_2

d_1 = first distance from source

d_2 = second distance from source

Note that since this equation is a simple ratio, units are not specified but must be consistent. Also, this is a point source approximation so estimates up close to the source will not be accurate.

Problem: A source emits particles (i.e., photons) that are measured at 250 particles/cm^2-sec at a distance of 1 meter. What activity will be detected at 2 meters?

Solution:

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$$I_2 = I_1\left(\frac{d_1}{d_2}\right)^2 = 250 \text{ particles/cm}^2\text{-sec}\left(\frac{1 \text{ m}}{2 \text{ m}}\right)^2 = 62.5 \text{ particles/cm}^2$$

10.1.2 Gamma Radiation Exposure

The roentgen value at 1 foot from a gamma emitter is described as:

$$S \cong 6CE \tag{257}$$

where

S = roentgens, per hour at 1 ft

6 = a constant for English units

C = curie strength of gamma emitter, Ci

E = energy of gamma radiation, MeV

Problem: Assume Iodine-131 emits gamma photons at different energies; one of which is 0.313 MeV. What is the partial exposure rate at 1 foot from a 10 mCi source due to this energy?

Solution:

$$S \cong 6CE \cong 6(10 \text{ mCi})(0.313 \text{ MeV}) = 18.8 \text{ mR/hr}$$

Note: This equation has an accuracy of about 20% between 0.07 and 4 MeV; that is why the symbol $\cong$ is used as it indicates "approximately equal to."

The following equation can be used to calculate the exposure rate from a gamma radiation source located some distance away.

$$D = \frac{\Gamma A}{d^2} \tag{258}$$

where

D = exposure rate, R/hour

Γ = gamma ray constant, R/mCi-hr

A = source activity, mCi

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d = distance from emitter, cm

Problem: Determine the exposure rate 1 meter from a 10 mCi source of Iodine-131. Assume the Gamma value for I-131 is Γ =2.18 R/mCi-hr at 1 cm.

Solution:

$$D = \frac{\Gamma A}{d^2} = \frac{(2.18 \text{ R/mCi-hr})(10 \text{ mCi})}{(100 \text{ cm})^2} = 0.00218 \text{ R/hr}$$

10.1.3 Equivalent Dose

The following equation converts an absorbed source in units of rad, to an equivalent dose in rem.

$$rem = (rad)(QF) \tag{259}$$

where

rem = equivalent dose, rem

rad = absorbed dose, rad

QF = quality factor that converts rad to rem

Problem: A worker may be exposed to 5 rad of neutron radiation. According to the International Commission on Radiological Protection, the Quality Factor (QF) for neutrons is 10. Calculate the worker's potential exposure.

Solution:

$$rem = (rad)(QF) = (5 \text{ rad})(10) = 50 \text{ rem}$$

10.1.4 Radioactive Decay

Radioactive elements can be characterized by a half-life, which is the time required to lose half its radioactive atoms.

This form of the radioactive decay equations can be used to determine the remaining residual activity in a body after a know exposure (amount and time).

$$A = A_i (0.5)^{\frac{t}{T_{1/2}}} \tag{260}$$

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where

A = radioactivity remaining after some time, mCi (or other appropriate units)

A_i = initial radioactivity, mCi (or other appropriate units)

t = elapsed time, units to match $T_{1/2}$

$T_{1/2}$ = half life, units to match t

Problem: 1.5 mCi of Iodine-123 (I-123) is used to image thyroid cancer. If I-123 has a half-life of 13 hours; what radioactivity will remain in the patient after 8 hours?

Solution:

$$A = A_i\left(0.5\right)^{\frac{t}{T_{1/2}}} = 1.5 \text{ mCi}\left(0.5\right)^{\frac{8\text{ hr}}{13\text{ hr}}} = 1.0 \text{ mCi}$$

Another form of the radioactive decay is:

$$A = A_i e^{\frac{-0.639t}{T_{1/2}}} \tag{261}$$

where

A = radioactivity remaining after some time, mCi (or other appropriate units)

A_i = initial radioactivity, mCi (or other appropriate units)

e = natural logarithm, 2.71828…

t = elapsed time, units to match $T_{1/2}$

$T_{1/2}$ = half life, min (or other appropriate units)

Problem: Recalculate the radioactivity from the previous problem using equation (261).

Solution:

$$A = A_i e^{\frac{-0.639t}{T_{1/2}}} = 1.5 \text{ mCi}^{\frac{-0.639(8\text{ hours})}{(13\text{ hours})}} = 1.0 \text{ mCi}$$

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10.1.5 Activity of a Radioactive Element

The activity remaining in a radioactive element can be calculated by the following equation:

$$A = \frac{0.693}{T_{1/2}} N_i \qquad (262)$$

where

A = radioactivity remaining after some time, mCi (or other appropriate units)

$T_{1/2}$ = half life, min (or other appropriate units)

N_i = the number of atoms

Problem: Calculate the activity (disintegrations per second) of 1 microgram of Iodine-123. I-123 has a half-life of 13 hours. The atomic weight of Iodine is 127.

Solution: First, we need to calculate the number of atoms in 1 microgram of I-123. This is accomplished using Avogadro's number; one mole of an element has 6.023 x 10^{23} atoms.

$$N = \frac{6.023 \text{ x } 10^{23}}{127} 1 \text{ x } 10^{-6} \text{ g} = 4.74 \text{ x } 10^{15} \text{ atoms}$$

We also need the half-life is seconds:

$$\text{T}_{1/2} = 13 \text{ hours} \frac{3600 \text{ sec}}{\text{hour}} = 46{,}800 \text{ sec}$$

Now, substituting into equation (262) yields:

$$A = \frac{0.693}{T_{1/2}} N_i = \frac{0.693}{46{,}800} 4.74 \text{ x } 10^{15} = 7.02 \text{ x } 10^{10} \text{ sec}^{-1}$$

Note that this is equivalent to 7.02x 10^{10} becquerel.

10.1.6 Radiation Attenuation by Layers

As radiation passes through some medium, energy is lost.

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The amount of radiation reduced as it passes through a number of half-layers is given by:

$$I = \left(\frac{1}{2}\right)^A I_o \qquad (263)$$

where

I = intensity of radiation leaving layer(s), mR/hour

I_o = original intensity of radiation striking layer(s), mR/hour

A = number of half-value layers, nondimensional

A similar expression applies to the number of tenth-layers.

$$I = \left(\frac{1}{10}\right)^B I_o \qquad (264)$$

where

I = intensity of radiation leaving layer(s), mR/hour

I_o = original intensity of radiation striking layer(s), mR/hour

B = number of tenth-value layers, nondimensional

Problem: A source of radiation has created a radiation intensity of 125 mR/hr. If six half-value layers (HVL) of a shielding material, each 0.5 inch thick, are provided, what is the reduced intensity in mR/hr?

Solution: For half-value layer calculations, we use equation (263)

$$I = \left(\frac{1}{2}\right)^A I_o = \left(\frac{1}{2}\right)^6 125 \text{ mR/hr} = 1.95 \text{ mR/hr}$$

Note that the thickness is not required for this solution; just the number of half-layers. By definition a half-layer will reduce the transmitted radiation by half. Also, similar calculations can be made with tenth-layer protection using equation (264).

The above two equation can be written in the following form, which simply replaces the A or B values (number of layers) with a term that calculates the number of layers based on the total thickness and the values of the partial (1/2 or 1/10) layer thicknesses (HVL and TVL, respectively).

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$$I = \frac{I_0}{2^{\frac{X}{HVL}}} \tag{265}$$

and

$$I = \frac{I_0}{10^{\frac{X}{TVL}}} \tag{266}$$

where

I = intensity of radiation leaving layer(s), mR/hour

I_o = original intensity of radiation striking layer(s), mR/hour

X = total thickness of layers, units to match HVL or TVL

HVL = thickness of half-value layers, units to match X

TVL = thickness of tenth-value layers, units to match X

Problem: A source of radiation has created a radiation intensity of 125 mR/hr. If 3 inches of a shielding material with a TVL of 1.5 inches are provided, what is the reduced intensity in mR/hr?

Solution: For a tenth-value layer calculation, we use equation (266):

$$I = \frac{I_0}{10^{\frac{X}{TVL}}} = \frac{125 \text{ mR/hr}}{10^{\frac{3}{1.5}}} = 1.25 \text{ mR/hr}$$

Equation (265) can be used for half-value layers.

If the incident and attenuated radiation, as well as the thickness of the half-value layer are known, the required thickness of a barrier medium can be found by re-arranging equation (265) and solving for X. This leads to:

$$X = 3.32 \log\left(\frac{I_0}{I}\right)(HVL) \tag{267}$$

A similar expression can also be found for the tenth-value layer problems by re-arranging equation (266).

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Problem: Derive equation (267).

Solution: We start with equation (265) and proceed as follows:

$$I = \frac{I_0}{2^{\frac{X}{HVL}}}$$

$$2^{\frac{X}{HVL}} = \frac{I_0}{I}$$

$$\log\left(2^{\frac{X}{HVL}}\right) = \log\left(\frac{I_0}{I}\right)$$

$$\log 2\left(\frac{X}{HVL}\right) = \log\left(\frac{I_0}{I}\right)$$

$$X = \frac{1}{\log 2}\cdot\log\left(\frac{I_0}{I}\right)HVL$$

$$X = 3.32\cdot\log\left(\frac{I_0}{I}\right)HVL$$

10.1.7 Exponential Rate Attenuation

As a medium thickness increases, the attenuation increases and can be written (with and without a buildup factor) as:

$$I = I_o B e^{-\mu x} \tag{268}$$

and

$$I = I_o e^{-\mu x} \tag{269}$$

where

I = attenuated radiation exposure rate, counts/min

I_o = original radiation exposure rate, counts/min

B = buildup factor, nondimensional

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e = natural logarithm, 2.71828…

μ = linear attenuation coefficient, cm^{-1}

x = thickness of attenuator, cm

Problem: Calculate the attenuation of radiation passing through a lead shield that is 2 cm thick. Assume a linear attenuation coefficient of 0.78 cm^{-1} and a buildup factor of 1.87.

Solution: Re-arranging equation (268) leads to:

$$\frac{I}{I_o} = Be^{-\mu x} = 1.87e^{-0.78cm^{-1}2cm} = 0.39 = 39\%$$

Since I is 39% of I_o, the attenuation is 61%.

10.1.8 Effective Half-Life

The rate at which radioactivity decreases in the body can be described by the effective half-life, which is a function of the biological half-life and radiological half-life, by the following expression:

$$\frac{1}{T_{1/2eff}} = \frac{1}{T_{1/2rad}} + \frac{1}{T_{1/2bio}} \qquad (270)$$

where

$T_{1/2eff}$ = effective half-life

$T_{1/2rad}$ = effective radiological half-life

$T_{1/2bio}$ = effective biological half-life

Note: Use same units for all three half-lives.

This equation can be re-arranged to provide the following form:

$$T_{1/2eff} = \frac{(T_{1/2rad})(T_{1/2bio})}{T_{1/2rad} + T_{1/2bio}} \qquad (271)$$

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Problem: Iodine-123 (I-123) has a half-life of 13 hours and a biological half-life of 120 days (2880 hours). What is the effective half-life of I-123?

Solution:

$$T_{1/2eff} = \frac{(T_{1/2rad})(T_{1/2bio})}{T_{1/2rad} + T_{1/2bio}} = \frac{(2880)(13)}{2880+13} = 12.9 \text{ hours}$$

The biological half-life of I-123 is long compared to the radiological half-life, so it does not contribute significantly to the effective half-life.

10.2 Non-Ionizing

Non-ionizing radiation has insufficient energy to ionize matter. The range of non-ionizing radiation includes lasers, ultraviolet (UV), visible, infrared (IR), radio-frequency (RF) and extremely-low frequencies (ELF).

10.2.1 Absolute Gain (Antenna)

The absolute gain equation simply converts the gain for a particular antenna into an absolute gain, as follows:

$$G = 10^{\frac{g}{10}} \tag{272}$$

where

G = absolute gain, nondimensional

g = gain for a particular antenna, dB

Problem: An indoor antenna has a power of 1 W and a gain of 2.3 dB; what is the antenna's absolute gain?

Solution: Using equation (272) and substituting the gain, we find:

$$G = 10^{\frac{g}{10}} = 10^{\frac{2.3}{10}} = 1.7$$

Note that the power is not required here and the Gain is nondimensional.

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10.2.2 Field Strength

The electric field strength can be converted to a power density with the following equation:

$$PD = \frac{E^2}{3770} \tag{273}$$

where

PD = electrical power density, mW/cm^2

E = electric field strength, V/m

3770 = conversion constant, ohms

Problem: What is the power density of an electric field with a strength of 250V/m?

Solution:

$$PD = \frac{E^2}{3770} = \frac{(250\,\mathrm{V/m})^2}{3770\,\Omega} = 16.6\,\mathrm{mW/cm^2}$$

Note: The omega symbol Ω is commonly used to indicate ohms.

The magnetic field strength can be converted to a power density with the following equation:

$$PD = 37.7H^2 \tag{274}$$

where

PD = magnetic power density, mW/cm^2

H = magnetic field strength, A/m

37.7 = conversion constant, ohms

Problem: What is the power density of a magnetic field with a strength of 1.5 A/m?

Solution:

$$PD = 37.7H^2 = 37.7\,\Omega(1.5\,\mathrm{A/m})^2 = 84.8\,\mathrm{m\,W/cm^2}$$

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For antennas, the far field power density can be calculates as follows:

$$W = \frac{GP}{4\pi r^2} = \frac{AP}{\lambda^2 r^2} \tag{275}$$

where

W = far field power density, W/m^2

G = gain

P = radiated power from antenna, W

π = 3.141593…

λ= wavelength, m (see equation (281))

r = distance from antenna, m

A = area of antenna, m^2

Problem: What is the power density 10 feet away from a 500 W radar transmitter that has an absolute gain of 2?

Solution: Converting 10 feet to meters (3.048 meters) and substituting the problem values into equation (275) leads to:

$$W = \frac{GP}{4\pi r^2} = \frac{(2)(500\,\mathrm{W})}{4\pi(3.048)^2} = 8.56\,\mathrm{W/m^2}$$

For antennas, the near field power density can be calculated as follows:

$$W = \frac{16P}{\pi D^2} \tag{276}$$

Note that the equation of the area of a circle is:

$$A = \frac{\pi D^2}{4} \tag{277}$$

So, for a dish-type antenna, equation (276) and (277) can be combined to find the near field power density as:

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$$W = \frac{4P}{A} \tag{278}$$

where

W = near field power density, W/m^2

P = radiated power from antenna, W

A = area of antenna, m^2

Problem: A round antenna with a diameter of 7 meter has a total feed input power of 112.2 Watts; what is the power density at the surface of the antenna?

Solution: Applying equation (278) leads to:

$$W = \frac{4P}{A} = \frac{4(112.2\,\mathrm{W})}{\left(\frac{\pi(7m)^2}{4}\right)} = 11.66\,\mathrm{W/m^2} = 1.166\,\mathrm{mW/cm^2}$$

Note: One W/m^2 is equal to 0.1 mW/cm.

10.2.3 Safe Distance for Non-Ionizing Radiation

An estimated safe distance from an antenna can be derived as follows:

$$r = \left(\frac{PG}{4\pi EL}\right)^{1/2} \tag{279}$$

where

r = distance, cm

P = emitted power, W

G = absolute gain, nondimensional

π = 3.141593…

EL = exposure limit, W/cm^2

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Problem: Calculate the distance at which the exposure level is exceeded for a 1000 W antenna with an absolute gain of 100. Assume an exposure limit of 10 W/m^2.

Solution: Substituting values into equation (279) leads to:

$$r = \left(\frac{PG}{4\pi EL}\right)^{1/2} = \left(\frac{(1000\,\mathrm{W})(100)}{4\pi\left(10\mathrm{W/m^2}\right)}\right)^{1/2} = 28.2\,\mathrm{m}$$

$$(28.2\text{ m})(3.28\text{ ft/m}) = 92.5\text{ ft}$$

10.2.4 Magnetic Flux Density

The following equation can be used to calculate the vector sum magnetic flux by taking the square root of the sum of the squares of measurements in the x, y, and z direction.

$$B_r = \sqrt{B_x^2 + B_y^2 + B_z^2} \tag{280}$$

where

B_r = resulting magnetic flux density, tesla

B_x = magnetic flux density in the x plane, tesla

B_y = magnetic flux density in the y plane, tesla

B_z = magnetic flux density in the z plane, tesla

Problem: Magnetic flux measurements are made in the x, y and z planes at a particular location and the following data recorded: B_x = 1.5 mT, B_y = 0.75 mT, B_z = 1.25 mT. Calculate the resulting magnitude of the magnetic flux.

Solution:

$$B_r = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(1.5\mathrm{mT})^2 + (0.75\mathrm{mT})^2 + (1.25\mathrm{mT})^2} = 2.1\,\mathrm{mT}$$

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10.2.5 Electromagnetic Radiation Wavelength and Frequency

Recall the wavelength and frequency relationship for sound moving through air; electromagnetic radiation behaves according to a similar relationship, except this equation is based on the speed of light (not the speed of sound).

$$c = \lambda f = \frac{\lambda}{T} \tag{281}$$

where

c = speed of light, $3x10^8$ m/sec

λ = wavelength, m

f = frequency, Hz

T = period, sec

Problem: A particular microwave oven operates with a wavelength of about 0.2m; what is its frequency?

Solution: Re-arranging equation (281) and substituting leads to:

$$f = \frac{c}{\lambda} = \frac{3 \text{ x } 10^8 \text{ m/s}}{0.2 \text{ m}} = 1.5 \text{ x } 10^9 \text{ s}^{-1} = 1500 \text{ MHz}$$

10.2.6 Effective Ultraviolet Irradiance

The effective irradiance from a broadband ultraviolet source can be calculated using the following expression:

$$E_{eff} = \sum E_\lambda S_\lambda \Delta_\lambda \tag{282}$$

where

E_{eff} = effective irradiance (relative to a source), W/m^2

E_λ = spectral irradiance, W/m^2-nm

S_λ = relative spectral effectiveness, nondimensional

Δ_λ = wavelength step, nm

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Note the summation sign (Σ) in equation (282). Remember that means the product of each $E_\lambda S_\lambda \Delta_\lambda$ term must be added to find the total effective irradiance.

Problem: A lamp has the following UV properties; calculate the effective UV irradiance.

Wavelength	Spectral Irradiance W/m^2-nm	Relative Spectral Effectiveness	Wavelength Step nm
254	0.01	0.5	1
300	0.03	0.3	1
315	0.1	0.003	1

Solution:

$$\begin{aligned} E_{eff} &= \sum E_\lambda S_\lambda \Delta_\lambda \\ &= \left(0.01 \text{ W/m}^2\text{-nm}\right)\left(0.5\right)\left(1\text{nm}\right) \\ &+ \left(0.03 \text{ W/m}^2\text{-nm}\right)\left(0.3\right)\left(1\text{nm}\right) \\ &+ \left(0.1 \text{ W/m}^2\text{-nm}\right)\left(0.003\right)\left(1\text{nm}\right) \\ &= 0.0143 \text{ W/m}^2 = 1.43 \text{ x}10^{-6} \text{ W/cm}^2 \end{aligned}$$

Note: The exposure time permitted for a given UV irradiance can be found using equation (291).

10.2.7 Lasers

10.2.7.1 Magnification

A laser that has been magnified will have a resulting irradiance that increases by the square of the magnification power, which can be written:

$$I = I_0 \cdot (magnification)^2 \tag{283}$$

where

I = irradiance after beam passes through magnifier, W/cm^2

I_o = irradiance prior to magnifier, W/cm^2

$magnification$ = the magnifying power

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Problem: What is the increase in irradiance of a 1 W/cm^2 laser beam passing through a 10x30 binocular lens?

Solution: It is possible to reduce the divergence of most lasers using simple optics. For example, a binocular lens will decrease the divergence by the magnification factor (e.g., 10x30 would reduce the divergence to 1/10th of its original divergence). The reduction in divergence will increase the irradiance per unit area according to equation (283). Therefore,

$$I = I_0 \cdot (magnification)^2 = \left(1 \text{ W/cm}^2\right)\left(10\right)^2 = 100 \text{ W/cm}^2$$

10.2.7.2 Optical Density

Protective eyewear for use around lasers is rated for optical density (OD), which is the attenuation factor by which the optical filter reduces beam power according to the following equation:

$$O.D. = \log \frac{I_o}{I} \tag{284}$$

where

$O.D.$ = optical density

I_o = irradiance prior to filter

I = irradiance after beam passes through filter

Note: For pulsed laser use J/cm^2; for CW lasers use W/cm^2

Problem: A pulsed laser produces a potential exposure of 2.6 x 10^{-2}J/cm^2. If the maximum permitted exposure level is 5.0 x 10^{-7}J/cm^2, calculate the optical density required to reduce the laser pulse below the permitted level.

Solution:

$$O.D. = \log \frac{I_o}{I} = O.D. = \log \frac{2.6 \text{ x } 10^{-2} \text{ J/cm}^2}{5.0 \text{ x } 10^{-7} \text{ J/cm}^2} = 4.72$$

10.2.7.3 Laser Beam Diameter

The diameter of a laser beam at some distance from the source can be estimated by:

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$$D_L = \sqrt{a^2 + \phi^2 r^2} \tag{285}$$

where

D_L = laser beam diameter at distance r, cm

a = emergent beam diameter, cm

ϕ = emergent beam divergence, radians

r = distance, cm

Problem: Determine the diameter of a laser beam 0.5 km away from a source with an emergent diameter of 2 cm and a beam divergence of 1 x10^{-4} radians.

Solution:

$$D_L = \sqrt{a^2 + \phi^2 r^2} = D_L = \sqrt{2^2 + \left(10^{-4}\right)^2 \left(5.0x10^4\right)^2} = 5.4 \text{ cm}$$

10.2.7.4 Nominal Hazard Zone (NHZ)

The safe use of lasers requires the evaluation of various safe distances. These are presented here.

The following equation can be used to calculate the distance from a laser at which the potential eye exposure no longer exceeds the permitted exposure limit.

$$r_{NHZ} = \frac{1}{\phi}\left(\frac{4\Phi}{\pi EL} - a^2\right)^{1/2} \tag{286}$$

where

r_{NHZ} = nominal hazard zone, cm

ϕ = emergent beam divergence, radians

Φ = total radiant power output of laser, W or J

π = 3.141593…

EL = exposure limit, W/cm^2 or J/cm^2

a = emergent beam diameter, cm

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Problem: Determine the hazard zone distance for a 0.2 J pulsed laser that has a beam divergence of 1 x10^{-3} radians and an emergent beam diameter of 0.6 cm. Assume the maximum permitted exposure level is 5.0 x 10^{-7}J/cm^2.

Solution:

$$r_{NHZ} = \frac{1}{\phi}\left(\frac{4\Phi}{\pi EL} - a^2\right)^{1/2} = \frac{1}{10^{-3}}\left(\frac{4(0.2)}{\pi(5\text{x}10^{-7})} - (0.6)^2\right)^{1/2} = 7.14\text{x}10^5\text{cm} = 7.14\text{km}$$

The following equation can be used to calculate the distance from a laser at which the potential eye exposure no longer exceeds the permitted exposure limit for the "lens on laser" case.

$$r_{NHZ} = \frac{f_o}{b_o}\left(\frac{4\Phi}{\pi EL}\right)^{1/2} \tag{287}$$

where

r_{NHZ} = nominal hazard zone, cm

f_o = focal length of lens, cm

b_o = diameter of laser beam incident on focusing lens, cm

Φ = total radiant power output of laser, W

π = 3.141593…

EL = exposure limit, W/cm^2

Problem: A 3000 W laser has a 12.7 cm focal length, and an incident beam diameter of 2.54 cm. Calculate the distance beyond which the irradiance is less than the permitted exposure level (assume 45 W/cm^2).

Solution:

$$r_{NHZ} = \frac{f_o}{b_o}\left(\frac{4\Phi}{\pi EL}\right)^{1/2} = \frac{12.7\,\text{cm}}{2.54\,\text{cm}}\left(\frac{4(3000\text{W})}{\pi(45\,\text{W/cm}^2)}\right)^{1/2} = 46\,\text{cm}$$

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The following equation can be used to calculate the distance from a laser at which the potential eye exposure no longer exceeds the permitted exposure limit when diffuse reflection is included.

$$r_{NHZ} = \left(\frac{\rho \Phi \cos \theta}{\pi EL} \right)^{1/2} \tag{288}$$

where

r_{NHZ} = nominal hazard zone, cm

ρ = effectiveness of diffuse reflecting surface, 100% = 1

Φ = total radiant power output of laser, W

θ = angle from normal for the viewing surface, degrees

π = 3.141593…

EL = exposure limit, W/cm^2

Problem: Calculate the nominal hazard zone distance of a 500 W laser. Assume 100% effective diffuse reflecting surface, a viewing angle of 0-degrees from normal, and an exposure limit of 0.05 W/cm^2.

Solution:

$$r_{NHZ} = \left(\frac{\rho \Phi \cos \theta}{\pi EL} \right)^{1/2} = \left(\frac{(1)(500\,\text{W})(\cos 0)}{\pi \left(0.05\text{W/cm}^2\right)} \right)^{1/2} = 56.4\,\text{cm}$$

10.2.7.5 Laser Barrier Distance

The following equation can be used to determine the minimum distance for a barrier to provide protection from a given laser.

$$D_s = \frac{1}{\phi} \left(\frac{4\Phi}{\pi TL} - a^2 \right)^{1/2} \tag{289}$$

where

D_s = separation distance for barrier, cm

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ϕ = emergent beam divergence, radians

Φ = total radiant power output of laser, W

π = 3.141593…

TL = threshold limit value for barrier, W/cm^2

a = emergent beam diameter, cm

Problem: A 400 W laser has a beam divergence of 2.5 x 10^{-3} radians, and an exit beam diameter of 0.5 cm. Calculate the barrier distance at which the irradiance is less than the worst case exposure level (assume 45 W/cm^2).

Solution:

$$D_s = \frac{1}{\phi}\left(\frac{4\Phi}{\pi TL} - a^2\right)^{1/2}$$

$$D_s = \frac{1}{2.5\text{x}10^{-3}}\left(\frac{4(400\text{W})}{\pi(45\text{W/cm}^2)} - (0.5\text{cm})^2\right)^{1/2} = 1330\,\text{cm} = 13.3\,\text{m}$$

10.2.8 Spatial Averaging of Measurements

Typically, multiple measurements of an electric or magnetic field strength are made so that an average value can be found. Typically ten or more measurements are required. The resulting field strength average is called the spatial average and is calculated as follows:

$$spatial\ average = \left(\frac{\sum_{i=1}^{N} FS_i^2}{N}\right)^{1/2} \qquad (290)$$

where

FS_i = field strength measurement i, V/m (electric) or A/m (magnetic)

i = incremental measurement count

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N = total number of measurements

Problem: Electric field strength measurements are made at ten locations and the following data recorded. What is the spatial average of the measurements?

Location	Field Strength (V/m)
1	10
2	10
3	12
4	14
5	16
6	20
7	18
8	14
9	12
10	8

Solution: The following table is setup to solve equation (290) for the data presented. Note that since there are ten samples, N=10.

Location	Field Strength (V/m)	FS^2
1	10	100
2	10	100
3	12	144
4	14	196
5	16	256
6	20	400
7	18	324
8	14	196
9	12	144
10	8	64
	$\sum_{i=1}^{N} FS_i^2$	1924
	$\dfrac{\sum_{i=1}^{N} FS_i^2}{N}$	192.4
	$\left(\dfrac{\sum_{i=1}^{N} FS_i^2}{N}\right)^{1/2}$	14

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Therefore, the spatially averaged electric field strength is 14 V/m.

10.2.9 Time Exposure for Non-Ionizing Radiation

The permissible exposure time in seconds, for exposure to ultraviolet radiation incident upon the unprotected eye or skin, may be computed by:

$$t = \frac{0.003\,\text{J/cm}^2}{E_{eff}} \tag{291}$$

where

t = exposure time, sec

E_{eff} = effective irradiance, W/cm^2

0.003 J/ cm^2 = 0.003 W-s/cm^2 = conversion factor, from effective irradiance to exposure time

Problem: A lamp used in an industrial process has an effective irradiance of 5.0 µW/cm^2. What is the permissible time exposure?

Solution:

$$t = \frac{0.003\,\text{J/cm}^2}{E_{eff}} = \frac{0.003\,\text{J/cm}^2}{5.0 \text{ x } 10^{-6}\ \text{W/cm}^2} = \text{ 600 seconds = 10 minutes}$$

Note: A Watt is also a Joule/second.

Exposure times to some type of non-ionizing radiation (e.g., radio frequency, microwave) are limited to a permissible level which is based on a six-minute exposure. When the actual exposure exceeds the allowable limit, the following equation can be used to determine an alternative exposure duration based on the actual exposure level.

$$t = \frac{EL}{ML} \text{ x } 0.1\,\text{hr} \tag{292}$$

where

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t = time (duration) of acceptable exposure to the actual exposure level, hr

EL = exposure limit, mW/cm^2

ML = measured (actual) level, mW/cm^2

0.1hr = 6 minutes; the basis for the permissible exposure limit

Problem: Assume that for incident electromagnetic energy frequencies between 10 MHz and 100 GHz, a Permissible Exposure Limit (PEL) of 10 mW/cm^2 (averaged over six-minute periods) has been specified. However, a worker is potentially subjected to 15 mW/cm^2. What is the acceptable exposure time?

Solution:

$$t = \frac{EL}{ML} \text{ x } 0.1\text{hr} = \frac{10 \text{ mW/cm}^2}{15 \text{ mW/cm}^2} \text{ x } 0.1\text{hr} = 0.067 \text{ hr} = 4 \text{ min}$$

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Electricity

11 Electricity

11.1 Ohm's Law

One of the fundamental laws of electrical circuits is Ohm's law. Ohm's law states that the current between two points in a conductor is directly proportional to the voltage across the two points, and inversely proportional to the resistance between them. Mathematically this can be written:

$$I = \frac{V}{R} \tag{293}$$

which can also be written

$$V = IR \tag{294}$$

where

V = the potential difference measured *across* the resistance, volts

I = the current through the resistance, amperes

R = the resistance of the conductor, ohms

Problem: A 120 volt power tool and long extension cord has a total equivalent resistance of 40 ohms. What is the current in the system?

Solution:

$$I = \frac{V}{R} = \frac{120\,\text{volts}}{40\,\text{ohms}} = 3\text{ amps}$$

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11.2 Joule's Law

Another important law that pertains to electrical circuits is Joule's law. Joule's law states that the rate of heat dissipation in a conductor is proportional to the square of the current through it and to its resistance. Mathematically this can be written:

$$P = I^2 R \tag{295}$$

where

P = power, watts

I = the current through the resistance, amperes

R = the resistance of the conductor, ohms

Equations (294) and (295) can be combined to yield:

$$P = IV \tag{296}$$

Problem: A forklift has lights that draw 5 amps each. Assuming a 12 Volt electrical system, what is the power to each light?

Solution:

$$P = IV = (5\,\text{amps})(12\,\text{volts}) = 60\,\text{watts}$$

11.3 Resistance

The electrical resistance of a conductor (R) can be calculated by the following equation:

$$R = \rho \frac{L}{A} \tag{297}$$

where

R = the resistance of the conductor, ohms

ρ = is the resistivity in units of ohm-feet

L = length of conductor, feet

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A = cross-sectional area of conductor, ft^2

Problem: What is the resistance in 1000 feet of 14 Gauge copper wire?

Solution: We can use equation (297) but some preliminary calculations are required first. From tables of properties for copper, we can find ρ_{copper} = 5.51E-08 Ohms-ft. Also, the cross-sectional area of the conductor must be found:

14 Gauge wire has a nominal diameter of 0.06408 inches, or 0.00534 ft

$$A = \frac{\pi d^2}{4} = \frac{\pi\left(0.00534\,\text{ft}\right)^2}{4} = 0.000022\text{ft}^2$$

Then substituting into equation (297) leads to:

$$R = \rho\frac{L}{A} = (5.51x10^{-8}\,\text{ohm-ft})\frac{1000\,\text{ft}}{0.000022\,\text{ft}^2} = 2.46\,\text{ohms}$$

Comparing this value to 2.53 ohms obtained from a wire data table shows about a 3% difference. This small difference can be attributed to the actual versus nominal dimension, as well as the resistivity of the actual copper used in the wire.

11.4 Equivalent Values for Components in Series and in Parallel

Whenever multiple resistors, capacitors, or inductors are located within the same electrical circuit, they can be reduced to a single equivalent part using the following equations. The resulting equivalent value depends on if the parts are in series, or in parallel.

11.4.1 Resistors in Series

$$R_{series} = R_1 + R_2 + \ldots + R_n \tag{298}$$

11.4.2 Resistors in Parallel

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \tag{299}$$

where

R = resistance, ohms

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11.4.3 Capacitors in Series

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n} \tag{300}$$

11.4.4 Capacitors in Parallel

$$C_{parallel} = C_1 + C_2 + \ldots + C_n \tag{301}$$

where

C = capacitance, farads

11.4.5 Inductors in Series

$$L_{series} = L_1 + L_2 + \ldots + L_n \tag{302}$$

11.4.6 Inductors in Parallel

$$\frac{1}{L_{parallel}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n} \tag{303}$$

where

L = inductance, henries

Problem: What is the equivalent resistance (in ohms) of three resistors, 1 ohm, 2 ohms and 3 ohms, in series, and in parallel?

Series Solution:

$$R_{series} = 1\,\text{ohms} + 2\,\text{ohms} + 3\,\text{ohms} = 6\,\text{ohms}$$

Parallel Solution:

$$\frac{1}{R_{parallel}} = \frac{1}{1\,\text{ohms}} + \frac{1}{2\,\text{ohms}} + \frac{1}{3\,\text{ohms}} = 1.833\,\text{ohms}^{-1}$$

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To solve for $R_{parallel}$ take the reciprocal, which leads to $R_{parallel}$ = 0.545 ohms

Note: The same approach that is used for resistors is used for inductors; that is the same math, just different units. However, capacitor formulas are “flipped” when compared to resistors and inductors.

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Ergonomics

12 Ergonomics

12.1 Revised NIOSH Lifting Equation

Per the *Applications Manual for the Revised NIOSH Lifting Equation* (1994), the recommended weight limit (RWL) is the principal product of the revised NIOSH lifting equation. The RWL is defined for a specific set of task conditions as the weight of the load that nearly all healthy workers could perform over a substantial period of time (e.g., up to 8 hours) without an increased risk of developing lifting-related lower back pain. The RWL is defined by the following equation:

$$RWL = LC \; x \, HM \; x \, VM \; x \, DM \; x \, AM \; x \, FM \; x \, CM \qquad (304)$$

where

RWL = recommended weight limit

LC = load constant

HM = horizontal multiplier

VM = vertical multiplier

DM = distance multiplier

AM = asymmetric multiplier

FM = frequency multiplier

CM = coupling multiplier

Substituting appropriate values from the *Applications Manual for the Revised NIOSH Lifting Equation*, equation (304) can be written:

$$RWL(lb) = (51)\left(\frac{10}{H}\right)\left[1-\left(0.0075\left|V-30\right|\right)\right]\left[0.82+\left(\frac{1.8}{D}\right)\right]\left(1-0.0032A\right)\left(FM\right)\left(CM\right) \qquad (305)$$

and

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$$RWL(kg) = (23)\left(\frac{25}{H}\right)\left[1-\left(0.003\left|V-75\right|\right)\right]\left[0.82+\left(\frac{4.5}{D}\right)\right](1-0.0032A)(FM)(CM) \quad (306)$$

where

H = horizontal location, measured from the mid-point of the line joining the inner ankle bones to a point projected on the floor directly below the mid-point of the hand grasps, inches (English) or cm (Metric)

V = vertical location, defined as the vertical height of the hands above the floor. V is measured vertically from the floor to the mid-point between the hand grasps, inches (English) or cm (Metric)

D = vertical travel distance, defined as the vertical travel distance of the hands between the origin and destination of the lift, inches (English) or cm (Metric)

A = asymmetric angle, defined as the angle between the asymmetry line and the mid-sagittal line, degrees

FM = frequency multiplier (see table below)

CM = coupling multiplier (see table below)

Coupling Multiplier (CM) Table

Coupling Type	Coupling Multiplier	
	V < 30 inches (75 cm)	V ≥ 30 inches (75 cm)
Good	1.00	1.00
Fair	0.95	1.00
Poor	0.90	0.90

Frequency Multiplier (FM) Table

Frequency Lifts/min (F)‡	Work Duration					
	≤ 1 Hour		> 1 but ≤ 2 Hours		> 2 but ≤ 8 Hours	
	V < 30 in.†	V ≥ 30 in.	V < 30 in.	V ≥ 30 in.	V < 30 in.	V ≥ 30 in.
≤ 0.2	1.00	1.00	0.95	0.95	0.85	0.85
0.5	0.97	0.97	0.92	0.92	0.81	0.81
1	0.94	0.94	0.88	0.88	0.75	0.75
2	0.91	0.91	0.84	0.84	0.65	0.65
3	0.88	0.88	0.79	0.79	0.55	0.55
4	0.84	0.84	0.72	0.72	0.45	0.45
5	0.80	0.80	0.60	0.60	0.35	0.35

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6	0.75	0.75	0.50	0.50	0.27	0.27
7	0.60	0.70	0.42	0.42	0.22	0.22
8	0.52	0.60	0.35	0.35	0.18	0.18
9	0.45	0.52	0.30	0.30	0.00	0.15
10	0.41	0.45	0.26	0.26	0.00	0.13
11	0.37	0.41	0.00	0.23	0.00	0.00
12	0.00	0.37	0.00	0.21	0.00	0.00
13	0.00	0.34	0.00	0.00	0.00	0.00
14	0.00	0.31	0.00	0.00	0.00	0.00
15	0.00	0.28	0.00	0.00	0.00	0.00
>15	0.00	0.00	0.00	0.00	0.00	0.00

† Values for V are inches.

‡ For lifting less frequently than once per five minutes, set F = 0.2 Lifts/min.

Problem: During his shift, a worker at a printing machine must occasionally lift a roll of paper stock and place it into the paper receiver. The rolls weigh 40 lbs each and are 30 inches in diameter and initially located on the floor. The grab points are the center of the roll, so the lifting point is 15 inches above the floor. The final placement height of the center of the roll is 63 inches above the floor. The horizontal distance from the roll's initial and final location is 23 inches. Calculate the recommended weight limit (RWL) for the original location of this task.

Solution: From the data given, and the tables above, we can determine the following multipliers:

- H = 23 inches
- V = 15 inches
- D = 48 inches
- A = 0 (assume no asymmetric movement)
- FM =1.0 (from frequency table footnote)
- CM =0.95 (from coupling table, assume the coupling is "fair")

Then using equation (305) for English units:

$$RWL(lb) = (51)\left(\frac{10}{H}\right)\left[1-\left(0.0075\left|V-30\right|\right)\right]\left[0.82+\left(\frac{1.8}{D}\right)\right](1-0.0032A)(FM)(CM)$$

$$RWL(lb) = (51)\left(\frac{10}{23}\right)\left[1-\left(0.0075\left|15-30\right|\right)\right]\left[0.82+\left(\frac{1.8}{48}\right)\right](1-0.0032(0))(1)(.95) = 16 \text{ lbs}$$

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12.1.1 Lifting Index

The Lifting Index (LI) provides a relative estimate of the physical stress associated with a manual lifting job. Per NIOSH, the lifting index may be used to identify potentially hazardous lifting jobs or to compare the relative severity of two jobs for the purpose of evaluating and redesigning them. From the NIOSH perspective, it is likely that lifting tasks with a LI > 1.0 pose an increased risk for lifting-related low back pain for some fraction of the workforce. Hence, the goal should be to design all lifting jobs to achieve a LI of 1.0 or less. The Lifting Index is calculated as follows:

$$LI = \frac{L}{RWL} \tag{307}$$

where

LI = Lifting Index

L = load weight

RWL = recommended weight limit, calculated using equations above

Note: In equation (307), any weight measure can be used, as long as L and RWL are in the same units.

Problem: Based on the data and the RWL calculated above, determine the Lifting Index for the task.

Solution:

$$LI = \frac{L}{RWL} = \frac{40 \text{ lbs}}{16 \text{ lbs}} = 2.5$$

Therefore, the actual load is nearly 2-1/2 times the recommended weight limit indicating this lifting task would be hazardous for a majority of healthy industrial workers.

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12.2 Heat Stress and Relative Humidity

12.2.1 Wet Bulb Globe Temperature

The Wet Bulb Globe Temperature (WBGT) is a composite temperature used to estimate the heat stress effect of temperature, humidity, wind speed, and solar heating on people. WBGT values are calculated by the following equations:

Indoor WBGT (or outdoors with no solar load)

$$WBGT = 0.7T_{WB} + 0.3T_{GT} \tag{308}$$

Outdoor WBGT (with a solar load)

$$WBGT = 0.7T_{WB} + 0.2T_{GT} + 0.1T_{DB} \tag{309}$$

where

$WBGT$ = wet bulb globe temperature, ^oF or ^oC

T_{WB} = wet-bulb temperature, ^oF or ^oC

T_{GT} = globe temperature, ^oF or ^oC

T_{DB} = dry-bulb temperature, ^oF or ^oC

For a description of these temperatures, see Section 14.

Problem: What is the wet bulb globe temperature (WBGT) at a sunny location if a wet bulb temperature is 85 ^oF, the globe temperature is 94 ^oF, and the dry blub temperature is 90 ^oF?

Solution: Since we are evaluating a sunny day, we use equation (309).

$$WBGT = 0.7T_{WB} + 0.2T_{GT} + 0.1T_{DB} = 0.7\left(85\ ^oF\right) + 0.2\left(94\ ^oF\right) + 0.1\left(90\ ^oF\right) = 87\ ^oF$$

Important: Note that when you do not include the solar load you do not simply drop the dry bulb measurement; the globe temperature multiplier is different. Compare equations (308) and (309).

Common psychrometric charts graphically illustrate the relationships between air temperature and relative humidity, as well as other properties of air. Psychrometric charts are versatile; by knowing just two properties of air, various

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other properties can quickly be determined. See Section 14 for more info and example of their use.

12.2.2 Heat Storage by Body

The thermal (heat) balance within a human body can be mathematically described as follows:

$$M - W = E + R + C + K + S \tag{310}$$

where

M = metabolic energy (heat) production, Btu/hr

W = external work rate, Btu/hr

E = evaporative heat change, Btu/hr

R = radiant heat change, Btu/hr

C = convective heat change, Btu/hr

K = conductive heat change, Btu/hr

S = energy (heat) storage rate by the body, Btu/hr

The term M-W is always positive.

The conductive heat change (K) is typically small and, if so, can be ignored. In such cases equation (310)can be written:

$$\Delta S = (M - W) \pm C \pm R - E \tag{311}$$

Note that as suggested by equation (311), the convective and radiative changes can be positive or negative (i.e., gains or losses). Evaporative changes are losses.

Assuming no net change in the storage of energy (heat) in the body (i.e., S=0), and no work is done (i.e., W=0) equation (311) can be rearranged to find the evaporative cooling required to offset the metabolic, convective and radiative changes:

$$E_{req} = M + C + R \tag{312}$$

where

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E_{req} = steady state evaporative heat loss, Btu/hr

Problem: A worker is conducting light work such that metabolic heat production minus the work expended is 650 Btu/hr. If the worker has a local fan that provides 50 Btu/hr of convective cooling, local equipment that causes a radiant heat gain of 125 Btu/hr, and an evaporative heat loss of 250 BTU/hr, what is the worker's net heat gain?

Solution:

$$\Delta S = (M - W) \pm C \pm R - E = (650) - 50 + 125 - 250 = 475 \text{ Btu/hr}$$

12.2.2.1 Convective Heat Gain/Loss by Body

The convective heat change can be calculated using the following equation:

$$C = 0.65 v^{0.6} (T_a - 95) \qquad (313)$$

where

C = convective heat change, Btu/hr

v = air velocity, ft/min

T_a = air temperature, ^{o}F

95 = mean weighted skin temperature, ^{o}F

Problem: What is the convective cooling of an outside worker who is exposed to a temperature of 75 ^{o}F and 15 mph winds?

Solution: Equation (313) uses an air speed in ft/min, so the wind speed must be converted from mph to ft/min.

$$C = 0.65 v^{0.6} (T_a - 95)$$

$$C = 0.65 \left(\frac{15 \text{ miles}}{\text{hr}} \frac{5280 \text{ ft}}{\text{mile}} \frac{1 \text{ hour}}{60 \text{ min}} \right)^{0.6} (75 - 95) = -969 \text{ Btu/hr}$$

12.2.2.2 Radiant Heat Gain/Loss by Body

The radiative heat change can be calculated using the following equation:

$$R = 15(T_r - 95) \qquad (314)$$

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where

R = radiative heat change, Btu/hr

15 = constant

T_r = mean radiant temperature, ^{o}F

95 = mean weighted skin temperature, ^{o}F

Problem: A worker is located in an area with production equipment that creates an average radiant environment of 110 ^{o}F. What is the radiant heat gain of a worker in this area?

Solution:

$$R = 15(T_r - 95) = 15(110 - 95) = 225 \text{ Btu/hr}$$

12.2.2.3 Maximum Evaporative Heat Loss

The maximum evaporative heat loss formula quantifies the amount of heat that is lost from the body through evaporative cooling.

$$E_{\max} = 2.4v^{0.6}\left(42 - vp_w\right) \tag{315}$$

where

E_{max} = evaporative heat loss, Btu/hr

2.4 = constant

v = air velocity, ft/min

42 = vapor pressure of water at 95 ^{o}F skin temperature, mmHg

vp_w = vapor pressure of water at ambient temperature, mmHg

Problem: What is the maximum evaporative loss of an outside worker who is exposed to a temperature of 75 ^{o}F and 15 mph winds?

Solution: Equation (315) requires the vapor pressure of water. Assuming an effective temperature that is between the body temperature and the ambient air, tables of water pressure can be consulted to find a water vapor pressure of 32 mmHg. Then applying equation (315) leads to:

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$$E_{\max} = 2.4v^{0.6}\left(42 - vp_w\right)$$

$$E_{\max} = 2.4\left(\frac{15 \text{ miles}}{\text{hr}}\frac{5280 \text{ ft}}{\text{mile}}\frac{1 \text{ hour}}{60 \text{ min}}\right)^{0.6}\left(42-32\right) = 1789 \text{ Btu/hr}$$

12.2.3 Heat Stress Index

The heat stress index (HSI) is one method of quantifying thermal stress. As can be seen in the following equation, it is simply the ratio of the steady state evaporative cooling to the maximum possible evaporative cooling, expressed as a percentage,

$$HSI = \frac{E_{req}}{E_{\max}} x100 \tag{316}$$

where

HSI = heat stress index, non dimensional

E_{req} = steady state evaporative heat loss, Btu/hr (see equation (312))

E_{max} = evaporative heat loss, Btu/hr

Problem: A worker has a maximum evaporative loss of 1789 Btu/hr. Calculate the Heat Stress Index (HSI) if a worker requires an evaporative heat loss of 475 Btu/hr.

Solution:

$$HSI = \frac{E_{req}}{E_{\max}} x100 = \frac{475 \text{ Btu/hr}}{1789 \text{ Btu/hr}} \text{ x } 100 = 27\%$$

12.2.4 Ventilation of Sensible Heat

Sensible heat is the heat which results in a temperature change only when a transfer takes place. For example, sensible heat is produced by a heating system or is removed by a refrigeration system. The volume of air required to dissipate the sensible heat load can be calculated in the following manner.

First, recalling from thermodynamics, the heat capacity of a system can be defined by:

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$$\dot{E} = \dot{m} c_p \Delta T \tag{317}$$

where

$\dot{E}$ = energy change in the system, Btu/min

$\dot{m}$ = mass rate of the system, lbs/min

c_p = specific heat of the system, Btu/lb-^{o}F (0.24 Btu/lb-^{o}F)

ΔT = change in temperature, ^{o}F

The mass flow rate can be found by:

$$\dot{m} = Q_s \cdot \rho_a \tag{318}$$

where

Q_s = volumetric flow rate of sensible air, ft^3/min

ρ_a = density of air, lb/ft^3 (0.075 lb/ft^3)

Combining equations (317) and (318) and defining $\dot{E}$ as the sensible heat, H_s, leads to:

$$H_s = Q_s \rho_a c_p \Delta T \cdot (60\text{min/hr}) \tag{319}$$

Note the 60 min/hr conversion is required since Qs has units of ft^3/min, and Hs has units of Btu/hr.

Rearranging to solve for Q_s, and substituting values for c_p and ρ_a, leads to:

$$Q_s = \frac{H_s}{1.08\,\Delta T} \tag{320}$$

This is sometimes written as:

$$cfm = \frac{\textit{Total Sensible Heat}\ \text{(Btu/hr)}}{1.08\left(\Delta T\right)} \tag{321}$$

Problem: Determine the volumetric air flow rate (cfm) required to limit an area with an industrial oven that produces 25,000 Btu/hr of heat to a 10 ^{o}F degree temperature rise.

Solution:

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$$cfm = \frac{\textit{Total Sensible Heat}\ (\text{Btu/hr})}{1.08(\Delta T)} = \frac{25{,}000\ \text{Btu/hr}}{1.08(10\ ^{\circ}\text{F})} = 2315\ \text{cfm}$$

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Statistical Tables

13 Statistical Tables

The following statistical tables are provided on the following pages:

- Area Under the Standard Normal Curve from 0 to Z
- Table of Percentage Points of the t Distribution
- Upper Percentage Points of the χ^2 Distribution

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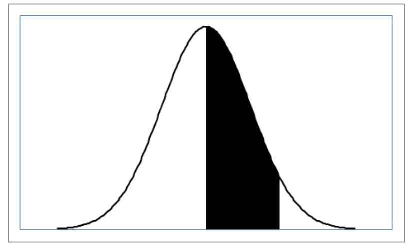
Area Under the Standard Normal Curve from 0 to Z



Z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

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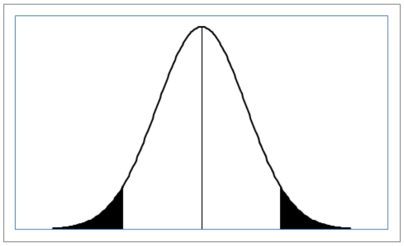
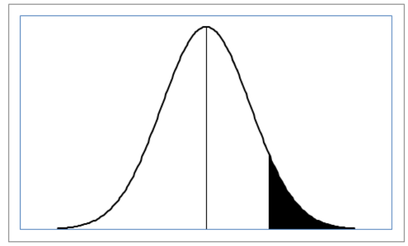
Table of Percentage Points of the t Distribution



Two-Sided



One-Sided

Degrees of Freedom	Probability				
	0.25 (one tail)	**0.1 (one tail)**	**0.05 (one tail)**	**0.025 (one tail)**	**0.01 (one tail)**
	0.50 (two tail)	**0.2 (two tail)**	**0.10 (two tail)**	**0.050 (two tail)**	**0.02 (two tail)**
1	1.000	3.078	6.314	12.706	31.821
2	0.816	1.886	2.920	4.303	6.965
3	0.765	1.638	2.353	3.182	4.541
4	0.741	1.533	2.132	2.776	3.747
5	0.727	1.476	2.015	2.571	3.365
6	0.718	1.440	1.943	2.447	3.143
7	0.711	1.415	1.895	2.365	2.998
8	0.706	1.397	1.860	2.306	2.896
9	0.703	1.383	1.833	2.262	2.821
10	0.700	1.372	1.812	2.228	2.764
15	0.691	1.341	1.753	2.131	2.602
20	0.687	1.325	1.725	2.086	2.528
25	0.684	1.316	1.708	2.060	2.485
30	0.683	1.310	1.697	2.042	2.457
∞	0.674	1.282	1.645	1.960	2.326

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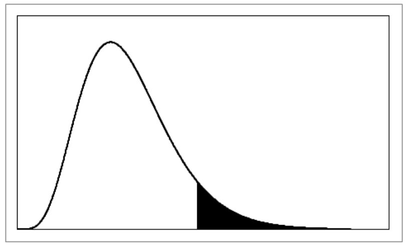
Upper Percentage Points of the χ^2 Distribution



Degrees of Freedom	Probability					
	0.99	**0.95**	**0.90**	**0.10**	**0.05**	**0.01**
1	0.000	0.004	0.016	2.706	3.841	6.635
2	0.020	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.345
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475
8	1.646	2.733	3.490	13.362	15.507	20.090
9	2.088	3.325	4.168	14.684	16.919	21.666
10	2.558	3.940	4.865	15.987	18.307	23.209
11	3.053	4.575	5.578	17.275	19.675	24.725
12	3.571	5.226	6.304	18.549	21.026	26.217
13	4.107	5.892	7.042	19.812	22.362	27.688
14	4.660	6.571	7.790	21.064	23.685	29.141
15	5.229	7.261	8.547	22.307	24.996	30.578
20	8.260	10.851	12.443	28.412	31.410	37.566
25	11.524	14.611	16.473	34.382	37.652	44.314
30	14.953	18.493	20.599	40.256	43.773	50.892

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Psychrometric Charts

14 Psychrometric Charts

Psychrometrics refers to the properties of gas-vapor mixtures, including air. Common psychrometric charts (see example below) graphically illustrate the relationships between air temperature and relative humidity as well as other properties of air. Psychrometric charts are versatile; by knowing just two properties of air, various other properties can quickly be determined.

14.1 Basic Definitions of Air

1. Atmospheric Air

   Atmospheric air is the air we breathe and use for normal ventilation. Air is primarily comprised of nitrogen and oxygen and small amounts of carbon dioxide, water vapor, and other gases. Miscellaneous contaminants such as dust, pollen, smoke, etc., may also be encountered depending on air quality.

2. Dry Air

   Dry air exists when all of the contaminants and water vapor have been removed from atmospheric air. By volume, dry air contains about 78 percent nitrogen, 21 percent oxygen, and 1 percent other gases. Dry air is used as the reference in psychrometrics.

3. Moist Air

   Moist air is a mixture of dry air and water vapor. Due to the variability of atmospheric air, the terms dry air and moist air are used in psychrometrics. For practical purposes, moist air and atmospheric air can be considered equal under the range of conditions normally encountered.

14.2 Basic Definitions of Air Temperature

1. Dry Bulb Temperature

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Dry bulb temperature is the air temperature determined by an ordinary thermometer. The dry bulb temperature scale is located at the base of the chart and the vertical lines indicate constant dry bulb temperature.

2. Wet Bulb Temperature

Wet bulb temperature reflects the cooling effect of evaporating water. Wet bulb temperature can be determined by passing air over a thermometer that has been wrapped with a small amount of moist cloth. The cooling effect of the evaporating water causes a lower temperature compared to the dry bulb air temperature. The wet bulb temperature scale is located along the curved upper left portion of the chart. The sloping lines indicate equal wet bulb temperatures.

3. Globe Temperature

Globe temperature is a measure of the radiant and convective temperature and is usually measured with what it known as a globe (or black globe) thermometer. This is a normal dry bulb thermometer encased in a 150mm diameter matte-black copper sphere whose absorptivity approaches that of the skin.

4. Dew Point Temperature

Dew point temperature is the temperature below which moisture will condense out of air. Air that is holding as much water vapor as possible is saturated, or at its dew point. Water will condense on a surface that is at or below the dew point temperature of the air. The dew point temperature scale is located along the same curved portion of the chart as the wet bulb temperature scale. However, horizontal lines indicate equal dew point temperatures.

14.3 Relative Humidity

Relative humidity is a measure of how much moisture is present compared to how much moisture the air could hold at that temperature. Relative humidity is expressed as a percent. Lines of equal relative humidity curve from the lower left to the upper right of the psychrometric chart. The 100 percent relative humidity (saturation) line corresponds to the wet bulb and the dew point temperature scale line. The line for zero percent relative humidity falls along the dry bulb temperature scale line.

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Problem: Air is known to be at 80°F (dry bulb) and 50 percent relative humidity. What are the wet bulb and dew point temperatures of this air?

Solution: First, locate the intersection of the 80°F dry bulb temperature line and the 50 percent relative humidity curve. From this intersection, follow the appropriate lines to the correct scales and find:

1. Wet bulb temperature = 67°F
2. Dew point temperature = 59°F

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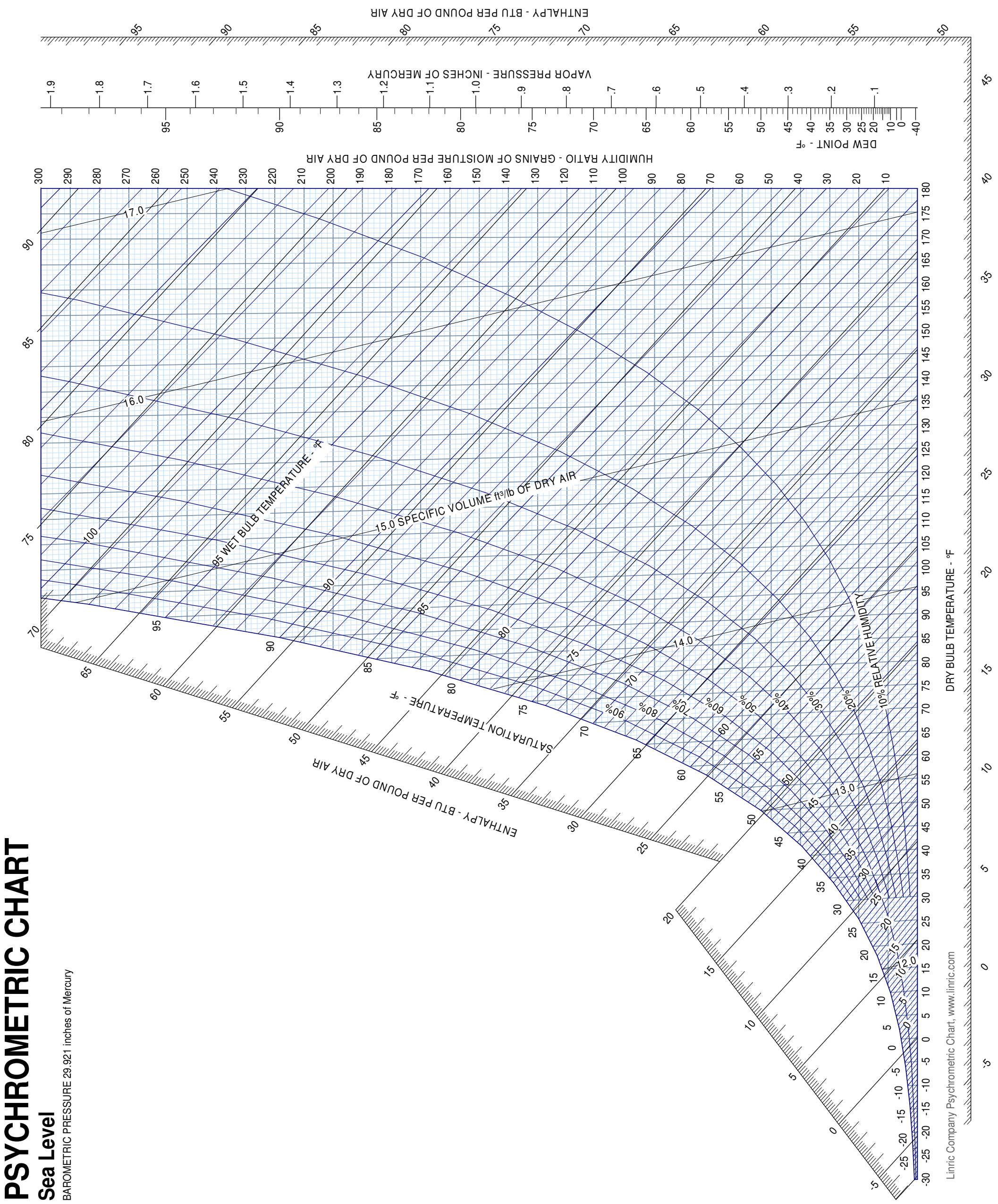

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Constant and Conversion

15 Constants and Conversions

15.1 Length

1 inch = 2.54 cm
1 foot = 30.48 cm = 0.3048 m
1 meter = 3.28 ft
1 mile = 5,280 ft
1 micron = 10^{-4} cm

15.2 Volume

1 molar volume = 1 gram-mole (at 0°C and 1 atm) = 22.4 L
1 molar volume = 1 gram-mole (at 25°C and 1 atm) = 24.45 L
1 ft^3 = 28.32 L = 7.481 U.S. gal = 0.0283 m^3
1 m^3 = 35.31 ft^3
1 L = 1.0566 qt = 61.02 in^3 = 0.03531 ft^3

15.3 Weight & Mass

1 lb = 453.6 grams
1 kg = 2.2 lb
1 gram = 15.43 grains

15.4 Pressure

1 atm = 14.7 psi = 760 mm Hg = 29.92 in. Hg = 33.93 ft water = 406.78 in. water = 1013.25 mbar = 101,325 pascals = 760 torr

15.5 Temperature

°F = 9/5(°C) + 32
°C = (°F - 32)/1.8
°R = °F + 460
K = °C + 273

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15.6 Angles

1 radian = $180^{o}/\pi$

15.7 Density of Water

density of water = 1 gram/cm^3 = 1.94 slugs/ft^3
weight density of water = 62.4 lb/ft^3

15.8 Density of Air

density of air (at 0°C and 1 atm) = 0.29 g/L
density of air (at 20°C and 1 atm) = 1.204 kg/m^3
density of air (at 70^{o}F and 1 atm) = 0.075 lb/ft^3

15.9 Energy

1 BTU = 1054.8 joules = 0.293 watt-hr
1 gram-calorie = 4.184 joules
1 faraday = 9.65 x 10^4 coulombs
1 watt = 1 joule/sec = 1 ampere x 1 volt
1 kwh = 3.6 x 10^6 joules

15.10 Radiation

1 becquerel = 1 disintegration/sec
1 currie = 3.7 x 10^{10} becquerel = 2.2x1012 dpm
1 rad = 10-2 gray (1 gray = 100 rad)
1 rem = 10-2 sievert (1 sievert = 100 rem)

15.11 Light

1 candela = 1 lumen/steradian
1 footcandle = 10.76 candela/m^2 = 10.76 lux

15.12 Magnetic Fields

1 tesla = 10,000 gauss

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15.13 Physical Constants

speed of sound in air (at 20°C) = 1125 ft/sec = 344 m/sec
speed of light = 3 x 10^8 m/sec
Planck's constant = 6.626 x 10^{-27} erg-sec = 6.626 x 10^{-34} joule-sec
Avogadro's number = 6.024 x 10^{23} /gram-mole
gas constant, R = 8.314 J/mole K = 0.082 L atm/mole-K = 10.731 ft^3-psi/°R-lb-mol
acceleration of gravity, g = 9.81 m/ sec^2 = 981 cm/sec^2 = 32 ft/sec^2

15.14 Standard Temperature and Pressure (STP)

STP (Physical Sciences) = 0°C at 1 atm
STP (Ventilation) = 70°F at 1 atm
STP (Industrial Hygiene) = 25°C at 1 atm

15.15 Miscellaneous

Effective area of filter, A_c = 385 mm for 25 mm filter

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Study Problems

16 Study Problems

The following study problems can be solved with formulas and information contained in this book. Solutions are provided in the following section. The sample problems focus on the mathematical skills required to solve all the formulas in this book, and others encountered in industrial hygiene and safety. The final examination, required for the issuance of CEUs, will be similar to those contained in this section, as well as those in other sections of this book. You should be comfortable solving these problems before requesting a final exam.

Problem 1

Calculate the volumetric flow rate in an 12-inch round flanged hood if the static pressure is 1.75 in. wc, the hood entry floss factor is 0.50 and the duct is moving air with a density factor or 0.95.

Problem 2

Calculate the combined sound pressure level created by four sources measured at 82 dB, 85 dB, 90 dB and 90 dB.

Problem 3

What volumetric flow rate is required in a 8 inch round plain duct hood located 1 foot from a location requiring a capture velocity of 150 fpm?

Problem 4

What is the equivalent capacitance (in farads) of three capacitors, 60µF, 40µF, and 12µF in series, and in parallel?

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Problem 5

At a hazardous materials laboratory, there are four HVAC charcoal filter units to remove airborne contaminants. If two are required to provide the required filter capacity, how many combinations of two filter units are provided by the set of four?

Problem 6

Indicate if each of the following logarithmic expressions is True or False.

$$\text{a)}\ \log_b x - \log_b y = \log_b \left(\frac{x}{y}\right)$$

$$\text{b)}\ \log_b \left(x^r\right) = r \log_b x$$

Problem 7

Calculate the attenuation of radiation passing through a lead shield that is 3 cm thick. Assume a linear attenuation coefficient of 0.78 cm^{-1} and a buildup factor of 1.87.

Problem 8

Simplify the following expression:

$$\left(-10z^3y^{-2}\right)^2\left(zy^4\right)^{-5}$$

Problem 9

As part of your annual budget, you need to allocate money to replace a piece of equipment that has an expected replacement cost of $20,000 in five years. How much would you have to place into the account each year for 5 years assuming an annual interest rate of 3%?

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Problem 10

A fan with an 8 inch impeller operates at 1500 RPM to supply 2000 cfm. If the impeller size and speed are changed to 6 inches and 2500 RPM, what will be the new flow?

Problem 11

Several water pressure measurements are taken and the following data are recorded: 75, 82, 101, 93, and 78; all is psi. Assuming a normal distribution, what is the probability of a reading greater than 110 psi? For the data set in the problem, assume the arithmetic mean and standard deviation are $\mu = \overline{X} = 85.8$, and $\sigma = 10.9$, respectively.

Problem 12

A forklift weighs 3980 lbs; what is its mass?

Problem 13

Electrical power to a factory fails an average of 7 times every year (e.g., storms, high winds, etc.). Calculate the reliability of the power system over a two-week period.

Problem 14

A shipping area has a ventilation system that provides 15 air changes per hour (ACH). What is the concentration of an airborne contaminant after 20 minutes if the initial concentration is 750 ppm and there is no additional contaminant?

Problem 15

Determine the Lifting Index (LI) for a task that has a RWL of 22.5 pounds and an actual lifted weight of 20 pounds. Also, what is indicated by the calculated LI?

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Problem 16

Calculate the distance at which the exposure level is exceeded for a 500 W antenna with an absolute gain of 20. Assume an exposure limit of 10 W/m^2.

Problem 17

A car is initially traveling at 20 mph and then accelerates at 30 $miles/hr^2$ for 1.5 miles. How fast is the car now going?

Problem 18

A duct lining material has an absorption coefficient of 0.35. Calculate the reduction in noise in a 6" by 12" duct.

Problem 19

Calculate the volumetric flow rate in an 8-inch round duct from a hood if the hood static pressure measurement is 2.0 in. wc and the hood entry coefficient is 0.72 (round duct, plain end).

Problem 20

A location has a barometric pressure of 29.10 mmHg and the temperature is 75 ^{o}F. What is the density correction factor for these conditions?

Problem 21

What is the TLV of a 25/75 mixture of hexane and xylene? Assume the TLV for hexane is 176 mg/m^3 and the TLV for xylene is 434 mg/m^3.

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Problem 22

Calculate the heat transfer rate through 4 inches of concrete when one surface is 212°F and the other is 70°F. Assume the thermal conductivity of the concrete is 0.45 Btu/hr-ft-°F

Problem 23

Determine the hazard zone distance for a 0.4 J pulsed laser that has a beam divergence of 1 x10^{-3} radians and an emergent beam diameter of 0.5 cm. Assume the maximum permitted exposure level is 5.0 x 10^{-7}J/cm^2.

Problem 24

A worker is exposed to toluene during their work. The TLV for toluene is 50 ppm. If the worker works 8.5 hours in a day, what is the permitted exposure to toluene?

Problem 25

What is the airborne concentration of asbestos fibers if 500 liters of air are sampled and the fiber density is 88 f/mm^2? Assume the effective area of the filter is 385 mm^2 (25 mm filter).

Problem 26

Calculate the equivalent sound pressure level for the following measurements: 85 dB for 3 hours, 90 dB for 2 hours, and 82 dB for 3 hours.

Problem 27

What is the power density of an electric field with a strength of 500V/m?

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Problem 28

Sound measurements record an average sound pressure level of 92 dB at a location 5 feet away from a compressor. What is the expected sound pressure level at 20 feet from the compressor?

Problem 29

Calculate the terminal settling velocity of 130 µm particles in still air. Assume the density of the particles is 1.15 g/cm^3. Also, the density of air is 0.0012 g/cm^3 and its viscosity is 0.000182 Poise. The gravitational acceleration is 980 cm/sec^2.

Problem 30

Ammonia has a chemical composition of NH_3 yielding a molecular weight of 17.03. Calculate its density in lbs/ft^3 at 0.95 atmospheres and 85 ^{o}F. Hint: See sample problem in Section 4.1.

Problem 31

Determine the diameter of a laser beam 1.0 km away from a source with an emergent diameter of 1 cm and a beam divergence of 1 x10^{-4} radians.

Problem 32

Several water pressure measurements are taken and the following data are recorded: 75, 82, 101, 93, and 78; all is psi. Calculate the standard deviation (n-1) for the data.

Problem 33

A box that weighs 225 lbs moves along a conveyor at 5 mph; what is its kinetic energy?

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Problem 34

A fault tree has an event C that will occur if event A occurs or if event A and B occurs. In Boolean algebra this can be shown as: $A + (A \cdot B)$. *Remember, in Boolean algebra "+" means OR and "·" means AND.*

If event A has a frequency of 1.2E-6 events/year, and event B has a frequency of 2.3E-6 event per year, calculate the frequency of event C.

Problem 35

What is the velocity in an air duct when the velocity pressure recorded is 1.20 in.wc? Assume standard air conditions.

Problem 36

Isopropyl alcohol (IPA) has a chemical formula of C_3H_8O, and therefore a molecular weight of 60. If a concentration of 500 ppm is measured, calculate the equivalent concentration in mg/m^3 of the IPA in air.

Problem 37

A source of radiation has created a radiation intensity of 500 mR/hr. If three tenth-value layers (TVL) of a shielding material are provided, what is the reduced intensity in mR/hr?

Problem 38

What is the convective cooling of an outside worker who is exposed to a temperature of 40 ^{o}F and 5 mph winds?

Problem 39

What is the friction loss when 500 gpm is flowing through 50 feet of 2 inch hose? Assume a Hazen-Williams coefficient of 130.

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Problem 40

Calculate the linear correlation coefficient for the following data set.

X	Y
3	2
4	3
6	7

Problem 41

Based on ACGIH requirements, what is the allowable exposure time for 82 dBA?

Problem 42

Calculate the hood entry loss factor for a hood with a velocity pressure of 1.50 in. wc and a hood entry loss of 0.85 in. wc.

Problem 43

Isopropyl alcohol (IPA) evolves at a rate of 1.5 cfm in a room that measures 25' x 45' x 9' high. If an initial concentration is measured at 50 ppm, what will the concentration of IPA be after 30 minutes of 2500 cfm of dilution air? Assume K=1 (i.e., Q'=Q).

Problem 44

The lower cutoff frequency of an octave band is 1.414 kHz. Calculate the upper cutoff frequency and the center frequency.

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Problem 45

Measurements made at two ends of a section of ductwork showed a total pressure at one location of 1.25 in. wc, and 0.95 in. wc at the other end. What is the head loss across the section of ductwork?

Problem 46

One pound of ethylene leaks from a cylinder into a room that measures 25 ft wide by 75 ft long by 9 feet high. Assume ethylene has a density of 0.0786 lbs/ft^3 at room temperature and pressure. What is the concentration in ppm (assume uniform mixing and no losses)?

Problem 47

A 1-inch valve is opened at the base of a water storage tank. The surface of the water in the tank is 20 feet above the open valve. What is the velocity and volumetric flow rate of the water exiting the open valve?

Problem 48

1.25 mCi of Iodine-123 (I-123) is used to image thyroid cancer. If I-123 has a half-life of 13 hours; what radioactivity will remain in the patient after 5 hours?

Problem 49

Calculate the pH of a solution that has 2.5 grams of HNO_3 in 3.0 liters of solution. The molecular weight of HNO_3 is 63.01 g/mole.

Problem 50

Two 120 volt power tools have a combined resistance of 60 ohms. What is the current in the system?

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Solutions to Study Problems

17 Solutions to Study Problems

Solution 1

Use equation (198):

$$Q = 4005A\sqrt{\frac{SP_h}{df\,(1+F_h)}} = 4005\left[\frac{\pi}{4}\left(\frac{12}{12}\right)^2\right]\sqrt{\frac{1.75}{0.95(1+0.5)}} = 3486 \text{ cfm}$$

Solution 2

Use equation (228):

$$SPL_{total} = 10\log\left(10^{\frac{82}{10}} + 10^{\frac{85}{10}} + 10^{\frac{90}{10}} + 10^{\frac{90}{10}}\right) = 93.9 \text{ dB}$$

Solution 3

First, the area of the hood is required in ft^2:

$$A = \frac{\pi d^2}{4} = \frac{\pi(8/12)^2}{4} = 0.35 \text{ ft}^2$$

Next, re-arranging equation (181) and substituting the appropriate values provides:

$$Q = V\left(10x^2 + A\right) = 150\left(10(1.0)^2 + 0.35\right) = 1552.5 \text{ cfm}$$

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Solution 4

For capacitors in series use equation (300):

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n} = \frac{1}{60\mu\text{F}} + \frac{1}{40\mu\text{F}} + \frac{1}{12\mu\text{F}} = 0.125/\mu\text{F}$$

$$C_{series} = 8\ \mu\text{F}$$

For capacitors in parallel use equation (301):

$$C_{parallel} = C_1 + C_2 + \ldots + C_n = 60\ \mu\text{F} + 40\ \mu\text{F} + 12\mu\text{F} = 112\ \mu\text{F}$$

Solution 5

Use equation (68) with n = 4 and k = 2.

$$C_k^n = \frac{n!}{k!(n-k)!} = \frac{4!}{2!(4-2)!} = 6$$

Solution 6

Both are true. See Section 1.4

Solution 7

Re-arranging equation (268) leads to:

$$\frac{I}{I_o} = Be^{-\mu x} = 1.87e^{-0.78cm^{-1}3cm} = 0.18 = 18\%$$

Since I is 18% of I_o, the attenuation is 82%.

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Solution 8

Using the rules in section 1.3, simplify as follows:

$$\left(-10z^{3}y^{-2}\right)^{2}\left(zy^{4}\right)^{-5}=100z^{6}y^{-4}z^{-5}y^{-20}=100zy^{-24}=\frac{100z}{y^{24}}$$

Solution 9

Use equation (78) to find the answer to this question:

$$A=F\left[\frac{i}{(1+i)^{n}-1}\right]=\$20,000\left[\frac{0.03}{(1+0.03)^{5}-1}\right]=\$3,767$$

Solution 10

Use equation (221):

$$Q_2=Q_1\left(\frac{Size_2}{Size_1}\right)^{3}\left(\frac{RPM_2}{RPM_1}\right)=2000\left(\frac{6}{8}\right)^{3}\left(\frac{2500}{1500}\right)=1406\text{ cfm}$$

Solution 11

First calculate the z-score. The formula for the z-score is given in equation (58).

$$z=\frac{X-\mu}{\sigma}=\frac{110-85.8}{10.9}=2.22$$

Now, going to a z-score table (see Section 13), find the area under the curve from 0 to 2.22 is .4868. However, the value beyond z = 2.22 is desired, so subtract the z-score from 0.5 (i.e., ½ of 1) and the answer is:

$$0.5-0.4868=0.0132=1.32\%$$

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Solution 12

Re-arranging equation (108):

$$m = \frac{W}{g} = \frac{3980\,\text{lbs}}{32.2\,\text{ft/sec}^2} = 123.6\,\text{slugs}$$

Solution 13

Use equation (71):

$$R(t) = e^{-\lambda t} = e^{-\left(\frac{7\,\text{failures}}{52\,\text{weeks}}\right)2\,\text{weeks}} = 0.76$$

Based on this calculation, the power supply system has a reliability of only about 76% so there is about a 24% probability of electrical system failure in a two week period.

Solution 14

Use equation (217):

$$C = C_0 e^{-tN} = (750\text{ ppm})\,e^{-(20/60\text{ hr})(15\text{ ACH})} = 5.0\text{ ppm}$$

Solution 15

Use equation (307):

$$LI = \frac{L}{RWL} = \frac{20\text{ lbs}}{22.5\text{ lbs}} = 0.89$$

Therefore, the Lift Index indicates this lifting task would not be hazardous for a majority of healthy industrial workers.

Solution 16

Substituting values into equation (279) leads to:

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$$r = \left(\frac{PG}{4\pi EL} \right)^{1/2} = \left(\frac{(500\,\text{W})(20)}{4\pi\left(10\text{W/m}^2\right)} \right)^{1/2} = 8.9\,\text{m}$$

$$(8.9\text{ m})(3.28\text{ ft/m}) = 29.3\text{ ft}$$

Solution 17

Using equation (121) and solving for v:

$$v^2 = v_o{}^2 + 2as$$

$$v^2 = (20\text{ mi/hr})^2 + 2\left(30\text{ mi/hr}^2\right)\cdot 1.5\text{ mi} = 490\text{ mi}^2/\text{hr}^2$$
$$v^2 = \sqrt{490\text{ mi}^2/\text{hr}^2}$$
$$v = 22.1\text{ mph}$$

Solution 18

Equation (239) provides the solution in dB/ft, so substitute values into the equation to find:

$$NR = \frac{12.6P\alpha^{1.4}}{A} = \frac{12.6(2\cdot 6 + 2\cdot 12)(0.35)^{1.4}}{6\cdot 12} = 1.45\text{ dB/ft}$$

Solution 19

Use equation (196):

$$Q = 4005C_e A\sqrt{|SP_h|} = 4005(0.72)\left[\frac{\pi}{4}\left(\frac{8}{12}\right)^2\right]\sqrt{|2.0|} = 1423\text{ cfm}$$

Solution 20

Use equation (172):

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$$df = \left(\frac{530}{T+460}\right)\cdot\left(\frac{BP}{29.92}\right) = \left(\frac{530}{75+460}\right)\cdot\left(\frac{29.10}{29.92}\right) = 0.96$$

Solution 21

Substituting into equation (91) yields:

$$TLV_{mix} = \frac{1}{\frac{F_1}{TLV_1} + \frac{F_2}{TLV_2}} = \frac{1}{\frac{.25}{176\,\text{mg/m}^3} + \frac{.75}{434\,\text{mg/m}^3}} = 317.6\,\text{mg/m}^3$$

Solution 22

Use equation (150):

$$\frac{q}{A} = k\frac{(T_1 - T_2)}{(x_1 - x_2)} = \left(0.45\text{ Btu/hr-ft}^{\text{o}}\text{F}\right)\frac{\left(212-70^{\text{o}}\text{F}\right)}{\left(4/12\text{ ft}\right)} = 191.7\text{ Btu/hr-ft}^2$$

Solution 23

Use equation (286):

$$r_{NHZ} = \frac{1}{\phi}\left(\frac{4\Phi}{\pi EL} - a^2\right)^{1/2} = \frac{1}{10^{-3}}\left(\frac{4(0.4)}{\pi\left(5\text{x}10^{-7}\right)} - (0.5)^2\right)^{1/2} = 1.0\text{x}10^6\text{ cm} = 10\text{ km}$$

Solution 24

First calculate the reduction factor for one day based on the hours worked using equation (94):

$$RF_{day} = \frac{8}{h}x\frac{24-h}{16} = \frac{8}{8.5}x\frac{24-8.5}{16} = 0.91$$

Next, multiply the TLV by the reduction factor to determine the adjusted TLV:

$$TLV_{permitted-day} = 0.91(50\,\text{ppm}) = 45.5\,\text{ppm}$$

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Solution 25

Applying equation (101) and substituting leads to:

$$C_{asb} = \frac{EA_c}{1000V_s} = \frac{\left(88\,\text{f/mm}^2\right)\left(385\,\text{mm}^2\right)}{1000 \cdot 500\,\text{L}} = 0.068\,\text{fibers/mL}$$

Solution 26

Use equation (232):

$$L_{eq} = 10\log\frac{1}{T}\left(\sum_{i=1}^{N}\left(10^{\frac{L_i}{10}}t_i\right)\right)$$

$$L_{eq} = 10\log\frac{1}{T}\left(\sum_{i=1}^{N}\left(10^{\frac{85}{10}}\cdot 3 + 10^{\frac{90}{10}}\cdot 2 + 10^{\frac{82}{10}}\cdot 3\right)\right) = 86.3\text{ dB}$$

Solution 27

Use equation (273):

$$PD = \frac{E^2}{3770} = \frac{\left(500\,\text{V/m}\right)^2}{3770\,\Omega} = 66.3\,\text{mW/cm}^2$$

Solution 28

Use equation (227):

$$SPL_2 = SPL_1 + 20\log\left(\frac{d_1}{d_2}\right) = 92\text{ dB} + 20\log\left(\frac{5\text{ ft}}{20\text{ ft}}\right) = 80\text{ dB}$$

Solution 29

Substituting the given data into equation (105) leads to:

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$$V_{TS} = \frac{gd_p^2(\rho_p - \rho_a)}{18\eta} = \frac{\left(980\,\text{cm/sec}^2\right)\left(0.0130\,\text{cm}\right)^2 (1.15 - 0.0012\,\text{g/cm}^3)}{18\left(0.000182\,\text{g/cm-sec}\right)} = 58.1\,\text{cm/sec}$$

Solution 30

First, take equation (81) and multiply each side of the equation by the molecular weight (MW); to provide:

$$MW \cdot P \cdot Vol = MW \cdot n \cdot R \cdot T$$

This can be rearranged to:

$$MW \cdot P = \left(\frac{MW \cdot n}{Vol}\right) R \cdot T$$

The term $\left(\frac{MW \cdot n}{Vol}\right)$ is the density (ρ):

$$MW \cdot P = \rho \cdot R \cdot T$$

Which can be rearranged to solve for d:

$$\rho = \frac{MW \cdot P}{R \cdot T}$$

Selecting the appropriate value for R (based on the units desired) :

$$\rho = \frac{17.03 \cdot 0.95\,\text{atm}}{\left(0.73\,\text{ft}^3 \cdot \text{atm/lb mole} \cdot \text{R}\right) \cdot \left(460 + 85\text{F}\right)} = 0.041\ \text{lbs/ft}^3$$

Solution 31

Use equation (285):

$$D_L = \sqrt{a^2 + \phi^2 r^2} = D_L = \sqrt{1^2 + \left(10^{-4}\right)^2 \left(1.0x10^5\right)^2} = 10.0\,\text{cm}$$

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Solution 32

Use equation (45) to solve this problem. The following table assists with the calculation.

	x_i	$\bar{x} - x_i$	$(\bar{x} - x_i)^2$
n=5	75	10.8	116.6
	82	3.8	14.4
	101	-15.2	231.0
	93	-7.2	51.8
	78	7.8	60.8
Sum	429		474.8
$\bar{x}$	85.8		
$\sqrt{\dfrac{\sum_{i=1}^{n}(\bar{x}-x_i)^2}{n-1}}$			10.9

So, 10.9 psi is the standard deviation of this data.

Solution 33

First, convert weight in pounds to slugs, and speed in mph to ft/sec, and then use equation (118) to calculate kinetic energy.

$$K.E. = \frac{mv^2}{2} = \frac{\left(\dfrac{225\,\text{lbs}}{32.2\,\text{ft/sec}^2}\right)\left[\left(5\,\dfrac{\text{miles}}{\text{hr}}\right)\left(5280\,\dfrac{\text{ft}}{\text{mile}}\right)\left(\dfrac{1\text{hr}}{3600\,\text{sec}}\right)\right]^2}{2} = 187.9 \text{ ft-lbs}$$

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Solution 34

Apply the rules of Boolean algebra presented in section 1.7:

A + (A · B) = A, so event C has a frequency of 1.2E-06 events per year.

Solution 35

Since this is for air, use equation (170):

$$V = 4005\sqrt{VP} = 4005\sqrt{1.2} = 4387 \text{ cfm}$$

Solution 36

Re-arrange equation (88) as follows:

$$mg/m^3 = \frac{(ppm)(MW)}{24.45} = \frac{(500)(60)}{24.45} = 1227 \text{mg/m}^3$$

Solution 37

For tenth-value layer calculations, use equation (264):

$$I = \left(\frac{1}{10}\right)^B I_o = \left(\frac{1}{10}\right)^3 500 \text{ mR/hr} = 0.5 \text{ mR/hr}$$

Solution 38

Equation (313) uses an air speed in ft/min, so the wind speed must be converted from mph to ft/min.

$$C = 0.65v^{0.6}\left(T_a - 95\right)$$

$$C = 0.65\left(\frac{5 \text{ miles}}{\text{hr}}\frac{5280 \text{ ft}}{\text{mile}}\frac{1 \text{ hour}}{60 \text{ min}}\right)^{0.6}\left(40-95\right) = -1378 \text{ Btu/hr}$$

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This is a significant amount of cooling.

Solution 39

First, use equation (148) to calculate the friction loss per foot, and then multiply that by the total length.

$$P_d = \frac{4.52Q^{1.85}}{C^{1.85}d^{4.87}} = \frac{4.52(500\,\text{gal})^{1.85}}{(130)^{1.85}(2\,\text{in})^{4.87}} = 1.86\,\text{psi/ft}$$

$$P_{total} = (50\,\text{ft})(1.86\,\text{psi/ft}) = 93\,\text{psi}$$

Note: Generally you want to use the actual, not nominal, value of the pipe or hose diameter. Here the nominal value for the hose diameter is used since no actual diameter was specified.

Solution 40

Use equation (62); that requires the average of the X and Y values. These are easily found to be 4.33 and 4, respectively.

$x = X - \bar{X}$	$y = Y - \bar{Y}$	xy	x^2	y^2
-1.33	-2.00	2.67	1.78	4.00
-0.33	-1.00	0.33	0.11	1.00
1.67	3.00	5.00	2.78	9.00
$\sum$		8.00	4.67	14.00

$$r = \frac{\sum xy}{\sqrt{\left(\sum x^2\right)\left(\sum y^2\right)}} = \frac{8.0}{\sqrt{(4.67)(14.0)}} = 0.99$$

A linear correlation coefficient of 0.99 indicates a very strong positive relationship between the data. Note: Although this sample problem only uses three data pairs, the method is typically used for larger data sets.

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Solution 41

Since question is concerned with ACGIH requirements, equation (242) is the appropriate equation to use.

$$T = \frac{8}{2^{\frac{(L-85)}{3}}} = \frac{8}{2^{\frac{(82-85)}{3}}} = 16 \text{ hours}$$

Solution 42

Re-arranging equation (183) and substituting leads to:

$$F_h = \frac{VP_d}{h_e} = \frac{1.50 \text{ in.wc}}{0.85 \text{ in.wc}} = 1.76$$

Solution 43

Use equation (204), but the final concentration (C_2) is embedded in this form of the equation, so solve for C_2.

$$\ln\left(\frac{G - Q'C_2}{G - Q'C_1}\right) = -\frac{Q'}{V}(t_2 - t_1)$$

$$\ln\left(\frac{1.5 - 2500 \cdot C_2}{1.5 - 2500(0.000050)}\right) = -\frac{2500}{10125}(30-0)$$

$$\frac{1.5 - 2500 \cdot C_2}{1.5 - 2500(0.000050)} = e^{\left(-\frac{2500}{10125}(30-0)\right)}$$

$$\frac{1.5 - 2500 \cdot C_2}{1.375} = 0.00061$$

$$C_2 = \frac{(0.00061)(1.375) - 1.5}{-2500} = 0.0006 = 600 \text{ ppm}$$

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Solution 44

The upper cutoff frequency is given by equation (249):

$$f_2 = 2f_1 = 2\cdot(1.414\text{ kHz}) = 2.828\text{ kHz}$$

The center frequency is given by equation (251):

$$f_c = \sqrt{2}f_1 = \sqrt{2}\cdot 1.414\text{ kHz} = 2\text{ kHz}$$

Solution 45

Combining equations (160) and (161) provides:

$$TP_1 = TP_2 + h_L$$

$$h_L = TP_1 - TP_2 = (1.25\text{ in.wc}) - (0.95\text{ in.wc}) = 0.3\text{ in.wc}$$

Solution 46

First, the inverse of the density shows ethylene occupies 12.72 ft^3/lb. Then using equation (84) leads to:

$$ppm = \frac{V_{contam}}{V_{air}}x10^6 = \frac{12.72\,\text{ft}^3}{(25\,\text{ft})(75\,\text{ft})(9\,\text{ft})}x10^6 = 754\,\text{ppm}$$

Solution 47

Use equation (133) and substitute the value for the height and the gravitational acceleration (32.2 ft/sec^2) to find the velocity:

$$V = \sqrt{2gh_v} = \sqrt{(2)(32.2\,\text{ft/sec}^2)(20\,\text{ft})} = 35.9\,\text{ft/sec}$$

Next, find the area of the flow by the area of a circle:

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$$A = \frac{\pi d^2}{4} = \frac{\pi (1/12)^2}{4} = 0.00545 \text{ ft}^2$$

The volumetric flow is given by equation (135):

$$Q_1 = A \cdot V = \left(0.00545\text{ft}^2\right)\left(35.9\text{ft/sec}\right) = 0.196 \text{ ft}^3/\text{sec}$$

This can easily be converted to gpm, if desired.

Solution 48

Use equation (260):

$$A = A_i (0.5)^{\frac{t}{T_{1/2}}} = 1.25 \text{ mCi} (0.5)^{\frac{5 \text{ hr}}{13 \text{ hr}}} = 0.96 \text{ mCi}$$

Solution 49

First calculate the number of moles of HNO_3:

$$\frac{2.5 \text{ grams}}{63.01 \text{ grams/mole}} = 0.0397 \text{ moles}$$

Then calculate the molarity of the solution:

$$M = \frac{0.0397 \text{ moles}}{3.0 \text{ liters}} = 0.0132M$$

Finally, use equation (97) to find the pH:

$$pH = -\log_{10}\left[H^+\right] = -\log_{10}\left[0.0132\right] = 1.88$$

Solution 50

Use equation (293):

$$I = \frac{V}{R} = \frac{120 \text{ volts}}{60 \text{ ohms}} = 2 \text{ amps}$$

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